Internship report

Semi-classical quantization of strings: Wilson loops in the AdS/CFT correspondence

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Abstract

This internship report deals with the work I have achieved during my five-month internship in the Emmy Noether Group "Gauge from strings", Humboldt-Universität zu Berlin. After recalling some basics about string theory, I dive into the AdS/CFT correspondence and exhibit two particular cases: the duality of $\mathcal{N} = 4$ SYM with type IIB string theory in $AdS_5 \times S^5$, and the link between ABJM theory and type IIA string theory in $AdS_4 \times \mathbb{CP}^3$. In both cases (the second being an original result) I compute at strong coupling the one and two-loop cusp anomalous dimension of a lightlike cusped Wilson loop. The results are linked via the all-loop Bethe ansatz for AdS_4/CFT_3 proposed in [1] and a conjecture recently made in [2].
A paper partially based on this work was published at the end of the internship [3].

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Contents

Introduction

Since its early days, physics has always been a matter of unification. Newton unified celestial and terrestrial motion, Maxwell unified electricity and magnetism, Einstein unified time and space. But since the middle of last century, a bigger form of unification is at hand. Physicists have been trying to merge into a single theory two of science's most honourable achievements: Einstein's General Relativity and the Standard Model of particle physics. This is not an easy task, because renormalizability problems appear everytime we try to build a version of quantum gravity.

Originally intended as a theory explaining strong interactions, string theory emerged in the eighties, and could be today one of the most promising candidates towards a unified theory of particles and gravity.

String theory is based on one simple idea: instead of considering pointlike particles, one deals with one-dimensional string to be the building blocks of the world as we know it. As simple as it looks, this has tremendous consequences on the structure of the underlying field theory.

After the two superstrings revolutions (1984 & 1995), one of the most exciting discoveries in the field of string theory to this day remains the Maldacena conjecture (1997), also called AdS/CFT correspondence [4]. Through this claim, a bridge can be built between two really different worlds: the world of particle physics, with a gauge theory that has no gravity; and the world of strings, which possesses spin-2 particles like graviton, and thus gravity. The correspondence states that the gauge theory takes place on the boundary of an AdS space, where the string theory actually lives. This conjecture has lead to a significant number of publications figuring out its implications. To this day, it remains unproved, but the recent progress in the field of integrability may change this.

A really important observable in gauge theories is the Wilson loop. It is the base to build other gauge-invariant objects, and therefore, gives a really interesting insight into the gauge theory. According to the Maldacena conjecture, the expectation value of a Wilson loop can be mapped to a particular minimal string configuration [4]. Performing the computation on both sides can give evidence for the conjecture. As an example, many calculations were done to map the Wilson loop in $\mathcal{N} = 4$ SYM to a string living in $AdS_5 \times S^5$.

The report proceeds with the following structure :

Section 1 is a reminder about some basics in string theory: actions, constraints, quantization. In section 2, I present the AdS/CFT correspondence with more details and give two examples of its realisation, which are of interest for our work. In section 3, the actual original result will be presented. It consists in a calculation of the anomalous dimension for a cusped Wilson loop at one- and two-loop order. First, the case of $AdS_5 \times S^5$ is used as a motivation for the correspondence, through the work of [5], which is understood and reproduced. Then, we turn to the $AdS_4 \times CP^3$ case and the action of [6], which we use to expand the Lagrangian at quartic order, allowing us to compute the quantities and compare them to the $AdS_5 \times S^5$ ones through the conjecture done in [1] and [2].

Conventions

We will be using most of the time the $(-,+,\dots,+)$ signature for the metric.

Greek indices $\mu,\nu,\dots$ are D-dimensional, with in general $D=10$ or 26.
Latin indices $a,b,\dots$ are two-dimensional worldsheet indices.

$\eta_{\mu\nu}=\eta^{\mu\nu}=\text{diag}(-1,+1,\dots,+1)$ is the Minkowski metric.
$g_{\mu\nu}$ refers to an indefinite Lorentzian metric of the embedding space.

1 String theory: a basic how-to

In this section, I will describe the minimal amount of string theory formalism that is needed for later computations. It is just a glimpse in all the framework I have studied, and an even tinier one in the actual framework that has been developed since the 80s.
My main references for this work are [7, 8, 9, 10].

1.1 Fields and actions

The goal is to describe strings evolving in a D-dimensional space. We keep D generic for now. It is useful throughout string theory to build analogies with pointlike descriptions: a particle moving through space-time will be described by D scalar fields parametrized by the proper time of the particle: $X^\mu(\tau)$. Since a string also has a spatial extension, we must add a spatial parametrization through a spacelike coordinate σ. Thus, our string is described by:

$$X^\mu(\tau,\sigma)\quad \mu\in\{0,1,\dots,D-1\} \tag{1}$$

The parameters will sometimes be written $(\tau,\sigma)\equiv(\sigma^0,\sigma^1)$.

These fields are scalar[3] bosonic fields. The surface described by the $X^\mu(\tau,\sigma)$ is called the worldsheet, while the embedding space in D dimensions is refered to as the target space.
We must now give these fields dynamics. In analogy to the pointlike particle action $S=-m\int\sqrt{-\dot{X}^\mu\dot{X}^\nu g_{\mu\nu}}\mathrm{d}\tau$ [4] , we write the *Nambu-Goto action* for the string:

$$S_{NG}=-T\int \mathrm{d}^2\sigma\sqrt{-\gamma} \tag{2}$$

where $\gamma=\det\,\gamma_{ab}$ is the determinant of the pull-back metric on the worldsheet defined as:

$$\gamma_{ab}=\frac{\partial X^\mu}{\partial\sigma^a}\frac{\partial X^\nu}{\partial\sigma^b}g_{\mu\nu} \tag{3}$$

T is the string tension, which is often written $T=\frac{1}{2\pi\alpha'}$, with α' the so-called Regge slope. This formula is a compact form for a string action, but the square root might be really nasty in many cases (e.g. in path integral formalism), so we trade it with an equivalent form, namely the *Polyakov action*:

[3] With respect to the worldsheet coordinates transformations. This reflects the reparametrization invariance of these fields.

[4] We recover the classical free action in the limit $c\to\infty$.

$$S_P = -\frac{T}{2}\int \mathrm{d}^2\sigma\sqrt{-\gamma}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu g_{\mu\nu} \tag{4}$$

where now γ^{ab} is a dynamical field, the auxiliary metric of the world-sheet.

1.2 Symmetries and invariances

It can be shown that the Polyakov action enjoys:

- (in case of flat target space $g_{\mu\nu} = \eta_{\mu\nu}$) Invariance under Poincaré transformations of the fields: $X^\mu \to \Lambda^\mu{}_\nu X^\nu + c^\mu$.
- Reparametrization invariance under $(\tau, \sigma) \to (\tau'(\tau,\sigma), \sigma'(\tau,\sigma))$ for any invertible maps τ', σ'.
- Weyl invariance, namely invariance under

$$\begin{aligned}&\tau, \sigma \to \tau, \sigma\\ &X^\mu \to X^\mu\\ &\gamma^{ab}(\tau,\sigma) \to \Omega^2(\tau,\sigma)^2\gamma^{ab}\ .\end{aligned}$$

  Weyl invariance is a property of the two-dimensional world, thus making strings really special objects.

The importance of these invariances is that, as in electromagnetism, they allow us to remove some redundant degrees of freedom, namely gauge-fix the worldsheet metric, in order to simplify the formulas. As a matter of fact, it is always [5] possible to choose the components of the metric such that $\gamma_{ab} = e^{2\phi(\tau,\sigma)}\eta_{ab}$ [6]. This is called the conformal gauge[7], and will be very useful in the following.

A remark should be done here. These invariances come naturally if we think in terms of *degrees of freedom* (d.o.f.) of the system. Indeed, the reparametrization invariance is a direct consequence of the fact that we have a redundancy in our description: these coordinates (σ, τ) have no physical meaning whatsoever because the physics should not depend on them.

1.3 Light-cone gauge

The process of quantizing a stringy system is complex. We won't describe all the formalism, and will only give a way to do it that will be useful in our computations.

We start by defining the light-cone coordinates:

$$X^\pm = \frac{1}{\sqrt{2}}(X^0 \pm X^{D-1}) \tag{5}$$

[5] In some curved spaces (as in [11]), this choice must be modified if combined with light-cone gauge-fixing.

[6] This is done in the following way: through a conformal transformation, $g'^{\alpha\beta} = e^{2\chi}g^{\alpha\beta}$, which implies for the Ricci scalar the relation $\sqrt{g'}R' = \sqrt{g}(R - 2\Delta\chi)$. Solving $R - 2\Delta\chi = 0$, we can impose $R' = 0$, which implies, in 2D, a vanishing Riemann tensor.

[7] We did not use all our invariances. We can still set $\phi = 0$ and get Minkowski space on the worldsheet.

Next step consists in using the gauge freedom to write X^+ as

$$X^+ = x^+ + \alpha' p^+ \tau \tag{6}$$

which can be redefined through a time shift to $X^+ = \alpha' p^+ \tau$.

Then, we remark that the equations of motions and Virasoro constraints allows to solve entirely X^- in terms of the other fields X^i. As a matter of fact, if we make the split $X^- = X_L^-(\sigma^+) + X_R^-(\sigma^-)$, we get:

$$\partial_+ X_L^- = \frac{1}{\alpha' p^+} \sum_{i=1}^{D-2} \partial_+ X^i \partial_+ X^i \qquad \partial_+ X_R^- = \frac{1}{\alpha' p^+} \sum_{i=1}^{D-2} \partial_- X^i \partial_- X^i \tag{7}$$

Hence, we only have $D-2$ degrees of freedom left, which luckily are all space-like. Therefore, we can hope that, when quantizing, we will get no negative states at all. It is indeed the case. Therefore, after imposing light-cone gauge, we can try and quantize the theory by building creation and annihilation operator as in any QFT. The interesting result is that we can build a spin-2 state that mimics the behaviour of the graviton, making the string theory a coherent theory of *quantum gravity*.

The tricky point of this approach is that it seems to violate Lorentz invariance, since we removed coordinates. To check that it is preserved, the generators of Lorentz algebra have to be computed, and it must be checked that they obey the usual commutation relations. It can be shown, after long computations reviewed in [8, 9] that the Lorentz algebra holds if $D = 26$. This gives us the critical dimension of the bosonic string[8].

1.4 From strings to superstrings

So far we have only used bosonic coordinates. We did not talk about it, but there is a problem with this approach. Indeed, the fundamental state of such a theory is a tachyon, whose mass obeys $m^2 \propto -\frac{1}{\alpha'}$. A tachyon is problematic since it travels faster then light, and violates therefore causality, on which a consistent quantum field theory must be based. It means we are developing the theory around a maximum instead of a minimum.

It can be shown that the theory behaves properly (has a stable minimum and non-tachyonic vacuum state) if we use supersymmetry. In addition to the D bosonic degrees of freedom X^μ, we add D spinors ψ_A^μ, where A is a spinor index.

A naive way to introduce these fermions is to add a Dirac action term

$$S = -\frac{T}{2} \int d^2\sigma \left(\partial_a X^\mu \partial^a X_\mu - i \bar{\psi}^\mu \rho^a \partial_a \psi_\mu \right) \tag{8}$$

where ρ are 2D Dirac matrices. We notice then that this action is invariant up to a total derivative under the following supersymmetry transformations (ϵ is a constant spinor):

$$\delta X^\mu = \bar{\epsilon} \psi^\mu \qquad \delta \psi^\mu = -i \rho^a \partial_a X^\mu \epsilon \tag{9}$$

This naive attempt, however, suffers from flaws:

[8]This dimension can be found to be 26 also using other constraints on the system (vanishing of the operator anomalous dimension, BRST quantization,...), which stands for an important self-coherence test of the theory.

- The supersymmetry at work is a supersymmetry of the worldsheet, which must be introduced by hand, and is therefore not really natural. This is the Ramond-Neveu formulation.

- There's a problem in counting the degrees of freedom. Indeed, we know from SUSY that the number of on-shell fermionic d.o.f. must match the bosonic one. However, if we choose Majorana-Weyl spinors, we are left with twice as many fermionic dof as bosonic ones. Therefore, we must require a new symmetry in order to reduce the number of fermionic dof. This is the so-called κ-symmetry.

There are a lot of technicalities in these problems, but the final answer is to use the framework of superspace and to make supersymmetry a manifest property of the target space: this is the Green-Schwarz approach. We can again impose a definite value of the space-time dimension to satisfy Lorentz algebra and preserve causality: $D = 10$. In this case, we are allowed to use Majorana-Weyl spinors.

The most general action we can get in this framework is the *Green-Schwarz action* whose derivation is really involved. For interest see Appendix B of [12] and chapter 4 and 5 of [8]. We will only be interested in the precise form of this action in definite cases as $AdS_5 \times S^5$ or $AdS_4 \times \mathbb{CP}^3$, for which this action can more simply be found through the coset construction.

This ends our first part about generalities. A lot of them had to be swept under the rug, but we now have enough formalism to really study some modern string issues.

2 The AdS/CFT correspondence

2.1 Formulation

The AdS_n/CFT_{n-1} correspondence, also called gauge-gravity duality, was proposed in 1997 by Maldacena [4]. It states the existence of a correspondence between a gauge theory enjoying conformal invariance in dimension $n-1$, and a string/M-theory on a compactified space, where one of the factor is the n-dimensional Anti de Sitter space AdS_n, which is the embedded hypersurface

$$- x_0^2 + x_1^2 + \cdots + x_{n-2}^2 - x_{n-1}^2 = R^2. \tag{10}$$

Stated differently, the gauge theory "lives" on the boundary of the space described by the string theory[9]. It is remarkable, since it makes a connection between a theory with no gravity at all (the gauge side), and a theory which includes gravity (the string side). The precise way in how the observables are sent onto an other is beyond the scope of this report. We shall have interest in a particular link between observables at the end of this section.

The AdS space will be here parametrized using the Poincaré patch. We do not give its explicit form now, since it will have two different expressions in the particular cases of [5] and [6].

We will now give two examples of this correspondence.

2.2 AdS_5/CFT_4 correspondence

The AdS_5/CFT_4 correspondence is the most studied, known and tested version of AdS/CFT. A useful summarized description of the correspondence can be found in [13]. It relates a type IIB[10] superstring theory on $AdS_5 \times S^5$ with the $\mathcal{N} = 4$ Super Yang-Mills (SYM) theory in flat 4D space.

$\mathcal{N} = 4$ SYM theory is the natural supersymmetric extension of the Yang-Mills theory. The most simple way to derive it is to start from a $\mathcal{N} = 1$ supersymmetric Yang-Mills theory in 10 dimensions, and to perform a reduction of six coordinates.

The action is, in 10D:

$$S_{10} = \frac{1}{g_{10}^2} \int d^{10}x Tr \left\{ -\frac{1}{2} F_{MN} F^{MN} - i\bar{\Psi}\Gamma^M D_M \Psi \right\}. \tag{11}$$

The first term is the classical Lagrangian for Yang-Mills theory, with the field strength $F_{MN} = \partial_M A_N - \partial_N A_M - i[A_M, A_N]$. The second one looks like the massless[11] Dirac equation lagrangian, with 10-dimensional gamma matrices (representations of the Clifford algebra), and the covariant derivative $D_M \Psi = \partial_M \Psi - i[A_M, \Psi]$.

The dimensional reduction is performed via $\partial_M A_N = \partial_M \Psi = 0$ for $M \in \{4, \ldots, 9\}$, and additionally $A_{i+3} = \phi_i$ for $i \in \{1, \ldots, 6\}$. The coupling constant g_{YM} of the four-dimensional theory is g_{10} multiplied by the volume integrals which remain from the reduction procedure.

[9] This has been linked in many ways to 't Hooft and Susskind's holography principle.

[10] II stands for the number of supersymmetric charges ($\mathcal{N} = 2$), B for the particular chirality of this theory.

[11] The fact that the matter fields are massless is important for the conformal invariance of the theory.

$\mathcal{N} = 4$ SYM is just a toy model for theoretical physics, since it enjoys both supersymmetry and conformal invariance, leading to a superconformal group. In the case of $\mathcal{N} = 4$ SYM, this group is $\mathfrak{psu}(2,2|4)$. This mathematical fact is of importance for the correspondence. Indeed, consider the isomorphism:

$$AdS_5 \simeq {}^{SO(2,4)} \diagup {}_{SO(1,4)} \qquad S^5 \simeq {}^{SO(6)} \diagup {}_{SO(5)} \tag{12}$$

Therefore, our background is isomorphic to:

$$AdS_5 \times S^5 \simeq \frac{SO(2,4) \times SO(6)}{SO(1,4) \times SO(5)} \subset \frac{PSU(2,2|4)}{SO(1,4) \times SO(5)} \tag{13}$$

Therefore, using the supercoset construction [14], one can construct the action of $AdS_5 \times S^5$ using this coset. The fact that the string theory on this background and $\mathcal{N} = 4$ SYM share a common superconformal group is a necessary, but not sufficient condition to the correspondence. It helps us to see where to seek for other examples of correspondence.

A lot of observables can then be computed from $\mathcal{N} = 4$ SYM: anomalous dimensions of local operators, scattering amplitudes,... but this is not the purpose of this report. See [13] for more details.

A commonly studied limit of this theory is the so-called *planar limit*. In $\mathcal{N} = 4$ SYM, we have two parameters: the rank N of the gauge group $SU(N)$, and the gauge coupling g_{YM}. The planar limit consists in sending N to infinity while sending g_{YM} to zero, while maintaing their product finite. In that case, $\lambda = g_{YM}^2 N$ is called the *'t Hooft coupling*. Therefore, when proceeding to perturbative expansion, we will expand in powers of λ.

Note that the AdS/CFT correspondence gives the link between g_{YM} and the string coupling constant g_S as

$$g_{YM}^2 = 4\pi g_s. \tag{14}$$

2.2.1 An aside: planar limit in string theory

A slightly interesting remark can be made here about perturbative expansions in string theory. Indeed, let us have a look at what happens when we compute the scattering amplitude of two strings. References here are chapter 3 of [9], chapter 7 of [8] and chapter 6 of [7].

We have to sum over the different topologies the worldsheet can choose. In fact, depending on the number of “loops”, we can have the topology of a sphere, a torus, a two-handle torus,...

To implement the action of this topology, we must add to the action a term of the form [12]

$$S_{topo} = \gamma\chi = \frac{\gamma}{4\pi} \int d^2\sigma \sqrt{-g} R, \tag{15}$$

where χ is a term known as the Euler characteristic of the two-dimensional surface. It is the only possibly consistent kind of interaction one may add which satisfies Weyl invariance.

According to differential geometry, this term should not depend on the metric. In fact, the Gauss-Bonnet theorem simply states that

[12] We don't follow the literature which denotes γ as λ, since for us λ is already the 't Hooft coupling.

$$\chi = 2(1-g), \tag{16}$$

where g is the *genus* of the surface.

Therefore, when considering different topologies, we will write

$$\sum_g e^{-2\gamma(1-g)} \int \mathcal{D}[X,g] e^{-S_{string}}. \tag{17}$$

The string coupling constant is then defined [13] as $g_s = e^{\gamma}$. However, we know from AdS_5/CFT_4 that $g_s = \dfrac{g_{YM}^2}{4\pi} = \dfrac{\lambda}{4\pi N}$. Therefore, the topology of genus g will have a factor

$$\alpha_g = g_s^{-2(1-g)} \propto N^{2(1-g)}, \tag{18}$$

and therefore, for $N \to \infty$, we only have a contribution from the zero-genus surfaces, i.e. the sphere.[14]

2.3 AdS_4/CFT_3 correspondence

These form of AdS/CFT will be of great interest in the next section. It relates a certain type of M-theory on $AdS_4 \times \mathbb{C}^4 \diagup \mathbb{Z}_k$ with the three-dimensionnal ABJM theory on the gauge side. However, for simplicity, we will be working in certain limits [15] that allow to go from $\mathbb{C}^4/\mathbb{Z}_k$ to $S^7/\mathbb{Z}_k \simeq \mathbb{CP}^3$, and the M-theory is then a type IIA string theory. Therefore, we relate this string theory in $AdS_4 \times \mathbb{CP}^3$ to the ABJM theory, which we describe thereafter.

2.3.1 Chern-Simons and ABJ(M)

$\mathcal{N} = 6$ ABJM theory, proposed in 2008 in [16], is a Chern-Simons matter theory. A $\mathcal{N} = 2$ Chern-Simons theory is a topological theory in three dimensions, which is in many ways analogue to SYM. It is described through the action:

$$S_{CS} = \frac{k}{4\pi} \int d^3x \ \mathrm{Tr} \left\{ \varepsilon^{\mu\nu\rho} \left(A_\mu \partial_\nu A_\rho + \frac{2}{3} A_\mu A_\nu A_\rho \right) - \bar{\chi}\chi + 2D\sigma \right\} \tag{19}$$

where k is the level of the theory, and has to be quantized to an integer (for gauge invariance). $A_\mu = A_\mu^a t_a$ is the gauge field, D is an auxiliary scalar, σ an auxiliary scalar field, and $\chi, \bar{\chi}$ are two Dirac spinors.

As in the case of SYM, we can define the planar limit by taking N to infinity, k to infinity, and $\lambda = \frac{N}{k}$ fixed. We will work again at strong coupling, namely $\lambda >> 1$.

To construct ABJ theory, we take two Chern-Simons theories with opposite levels k and $-k$, altogether with two gauge groups $SU(N_1)$ and $SU(N_2)$. They are coupled to four matter fields $\Phi_{i=1,\ldots,4}$ which are hypermultiplets of the form

[13] It must be noted that γ, or equivalently g_s, is not arbitrary: it can be computed as the expectation value of a constant dilation field. See (7.14) in [7] for more details.

[14] The relation between the genus expansion of two-dimensional surfaces and the perturbative expansion of a $SU(N)$ gauge field theory in the large N limit is originally due to 't Hooft.

$$\Phi = \{\phi, \bar{\phi}, \psi, \bar{\psi}, F, \bar{F}\} \tag{20}$$

where ϕ is a complex scalar field, ψ a complex Dirac spinor, and F is an auxiliary complex field.

The ABJM theory corresponds to the case $N_1 = N_2$. A good review on the subject can be [17]. Note that though the Chern-Simons theory has been known for a long time, ABJM theory only came up recently, particularly through its interest in the AdS/CFT duality.

2.4 Wilson loops and gauge/gravity

The main interest of this correspondence for us will be in the computation of Wilson loops.

In a non-abelian gauge field theory, a Wilson loop is defined as:

$$W(\mathcal{C}) = \frac{1}{N} Tr \left\{ \mathcal{P} \left(\exp \ i \int_{\mathcal{C}} A_\mu dx^\mu \right) \right\} \tag{21}$$

where $A_\mu = A_\mu^a t_a$ is the gauge field, t_a are the generators of the gauge group $SU(N)$. $\mathcal{P}$ is the path ordering operator (spatial equivalent of time ordering operator). It is needed since the theory is non-abelian, hence one must order the generators t^a. $\mathcal{C}$ is a curve (not necessarily closed).

The Wilson loop is a non-local, gauge invariant observable. Indeed, if we take the simpler abelian case of $U(1)$, we can define a comparator between the points x and y to be

$$U_P(y,x) = \exp(-ie \int_P dx^\mu A_\mu) \tag{22}$$

If $A_\mu(x) \to A_\mu(x) - \frac{i}{e}\partial_\mu \alpha(x)$ under a gauge transformation, then $U_P(y,x) \to e^{i\alpha(y)} U_P(y,x) e^{-i\alpha(x)}$. Note that the expression of the comparator depends on the path P. If we take a closed curve $\mathcal{C}$ it becomes additionally gauge-invariant, thanks to the covariance of the comparator and the cyclicity of the trace.

It can be shown [18] that all gauge-invariant functions of A_μ are to be obtained by Wilson loops for different choices of $\mathcal{C}$. A notable example is the field strength $F_{\mu\nu}$.

The generalization to the non-abelian case is straightforward and yields expression (21).

We will see later that in the framework of involved theories, one can be interested in the coupling to the matter fields. In the cae of $\mathcal{N} = 4$ SYM, the Wilson loop admits a nice generalization

$$W(\mathcal{C}) = \frac{1}{N} Tr \ P \left\{ \exp \left[\oint_{\mathcal{C}} ds \left(i \frac{\mathrm{d}x^\mu}{\mathrm{d}s} A_\mu^a t^a + \Phi_I \Theta^I \left| \frac{\mathrm{d}x^\mu}{\mathrm{d}s} \right| \right) \right] \right\} \tag{23}$$

Here, Φ_I are the matter fields, and Θ^I is a "coupling" vector selecting which fields are coupled to the loop. The fields are taken in the adjoint representation of the gauge group.

Very intuitively, a Wilson loop can be understood in terms of Feynman diagrams. For example, in the non-abelian case of $SU(3)$, computing the Wilson loop can amount to computing the interacting potential of a quark-antiquark pair generated from the vacuum[19].

According to the AdS/CFT correspondence, the expectation value of a Wilson loop on the gauge side can be related to the partition function of a string configuration on the string side. This equivalence is made explicit by mapping, via an exponential map, the Wilson loop to a pair of antiparallel lines on the boundary of space.

The treatment of Wilson loops in large N gauge theories is performed in [20].

2.5 Semi-classical approach

This internship was all about the semi-classical quantization of strings. We would like to motivate shortly this kind of framework.

The form of the Green-Schwarz action seems really difficult to handle. Even after making a choice of coordinate, fixing the gauges (metric, κ-symmetry,...), choosing the vielbeins, its interpretation remains really complicated. The hope for directly quantizing such an action is to this day still a dream.

Therefore, we use semi-classical quantization: a particular solution of the equations of motion is chosen (it can be the trivial one), and then small fluctuations around this equilibrium are set on. We can now use all the techniques of perturbative framework, in particular evaluate path integrals using Feynman diagram techniques.

Some classical solutions to strings evolving in AdS are summed up in [19].

We have now the theoretical introduction to study a real string theory problem in the framework of AdS/CFT: the computation of the anomalous dimension of a cusped Wilson loop via a particular string configuration.

3 One- and two-loop anomalous dimension for cusped Wilson loops

In this section, we describe the research project. All calculations done here are in Euclidean space[15].

3.1 Geometry

Our goal is to study a cusped Wilson loop. A cusp can be in general described as a non-differentiable change of direction in the loop. It can be parametrized by an angle ϕ.

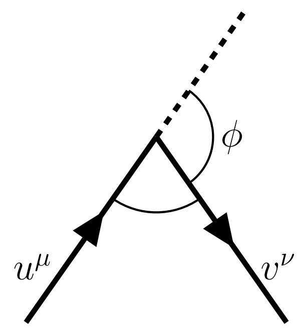

Figure 1: Cusped Wilson loop with angle cos $\phi = \frac{u_\mu v^\mu}{|u||v|}$ (figure from [12])

The natural quantity associated with a Wilson loop is its expectation value. As a matter of fact, when dealing with a cusped Wilson loop, we expect this quantity to be

$$\langle \mathcal{W} \rangle \propto e^{-\Gamma_{cusp}(\lambda,\phi) \log \frac{L_{IR}}{\epsilon_{UV}}} \tag{24}$$

where L_{IR} and ε_{UV} are infrared and ultraviolet cutoffs. The quantity we need to deal with is therefore the cusp anomalous dimension $\Gamma_{cusp}(\lambda, \phi)$, which is actually the coefficient of the regularized divergence. It has been linked to many other quantities (energy of large strings, scattering amplitudes in their infrared strucutre), therefore its computation is of highest interest.

At this point, AdS/CFT plays a role. We can compute the Wilson loop expectation value through the partition function of a string configuration. More precisely

$$\langle W_{cusp} \rangle = Z_{string} = \int \mathcal{D}[\phi] e^{-S_E(\phi)} \tag{25}$$

where S_E is the euclidean action of the string, depending on bosonic and fermionic fields ϕ. According to the correspondence, in order to map the cusped Wilson loop, one should consider a string whose ends lie on the boundary of AdS, and two extremities make a geometrical angle of $\pi - \phi$. See [19] and references therein.

[15] The time coordinate is Wick rotated according to $x^0 \to -ix^0$

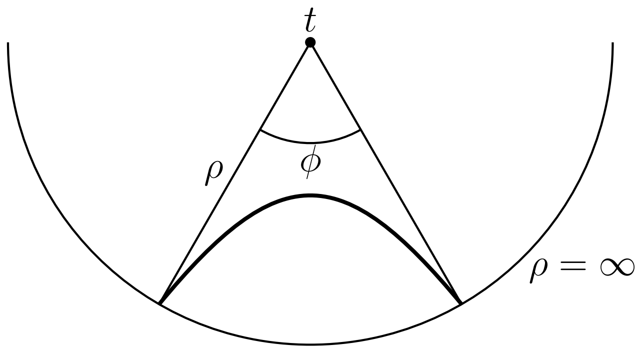

Figure 2: String configuration in AdS_4 (figure from [12])

We will describe a light-like cusp, obtained from the space-like one described above by taking the limit $\phi \to i\infty$. After doing this, $\Gamma_{cusp}(\lambda, \phi)$ becomes linear in ϕ

$$\Gamma_{cusp}(\lambda, \phi) \underset{\phi \to i\infty}{\sim} \frac{\phi}{2} f(\lambda), \tag{26}$$

where $f(\lambda)$ only depends on the 't Hooft coupling and is what in literature is referred to as (light-like) cusp anomaly, or simply scaling function.

Now, we need a classical background solution for this case. It turns out that the relevant solution is such that the Lagrangian of fluctuations over it has constant coefficients. Then the computation of Γ_{cusp} follows the expansion scheme described in [5]

$$Z_{string} = e^{-W} \Rightarrow W = W_0 + W_1 + W_2 + \cdots = \frac{1}{2} f(\lambda) V \tag{27}$$

where $V = \frac{1}{4} \int dt ds \equiv \frac{1}{4} V_2$ is the infinite worldsheet area and is the string counterpart of the divergence in (24).

Since in front of the string action a $\sqrt{\lambda}$ appears (see formula (35) below), which play the role of $1/\hbar$, the perturbative expansion above results in the following one for the cusp anomaly:

$$f(\lambda) = \sqrt{\lambda} \left(a_0 + \frac{a_1}{\sqrt{\lambda}} + \frac{a_2}{\lambda} + \dots \right) \tag{28}$$

The way to link the a_k constants and the partition function in path integral formalism is explained in [18]. We will be following these steps:

1. evaluate the action on the chosen classical solution. This gives the a_0 term.

2. expand the Lagrangian at quadratic order (there's no linear order, since we are expanding around a solution of the e.o.m.). It can be put in the form of a Gaussian integral, therefore, we compute the mass spectrum by identifying the free bosonic (Klein-Gordon) and fermionic (Dirac) kinetic and mass terms. After diagonalizing the propagators, we get

$$Z_{string} = \int \mathcal{D}[\phi_b, \phi_f] e^{-^t\phi_b K_b \phi_b - i^t\phi_f K_f \phi_f} \tag{29}$$

and therefore, $\mathcal{W}^{(1)}_{cusp} = -\ln Z_{string} = \ln\left(\frac{\det\ K_b}{\det\ K_f}\right)$, which finally gives back a_1.

3. the contribution of a_2 can be interpreted as coming from all connected Feynman diagrams with two loops. We shall therefore expand the Lagrangian at quartic order, deduce the vertex rules, study the contributing diagrams, and compute their contribution to $W^{(2)}_{cusp}$, from which we finally take a_2.

3.2 Computation in $AdS_5 \times S^5$

This section mainly follows what was done in [5]. We deal here with a string in $AdS_5 \times S^5$. The background is defined , for $m = 0, 1, 2, 3, M = 1, \ldots, 6$, as

$$ds^2 = z^{-2}(dx^m dx_m + dz^M dz^M) = z^{-2}(dx^m dx_m + dz^2) + du^M du^M \tag{30}$$

$$x^m x_m = x^+ x^- + x^* x \qquad x^\pm = x^3 \pm x^0 \qquad x = x^1 + ix^2 \tag{31}$$

$$z^M = zu^M \qquad u^M u^M = 1 \qquad z = (z^M z^M)^{1/2} = e^\phi \tag{32}$$

The AdS light-cone gauge in [11] completely fixes the gauge freedom:

- a “modified” conformal gauge for the auxiliary world-sheet metric

$$\sqrt{\gamma}\gamma^{ij} = \begin{pmatrix} z^2 & 0 \\ 0 & z^{-2} \end{pmatrix} \tag{33}$$

- bosonic light-cone gauge : $x^+ = x^2 + x^0 = \tau \qquad p^+ = 1$.

- fermionic light-cone gauge : $\Gamma^+\theta^I = 0$ where $\theta^I, I = 1, 2$ are the two Majorana-Weyl spinors of the model.

The open strings surface lands on a light-like cusp in the AdS boundary at $z = 0$ and oscillates around the null cusp background

$$z = e^{2\varphi} = \sqrt{\frac{\tau}{\sigma}} \quad x^1 = 0 \quad z^a = \bar{z}_a = 0 \quad \theta^A = \eta^A = \bar{\theta}_A = \bar{\eta}_A = 0 \tag{34}$$

The requirement that the string worldsheet described by these coordinates ends on a cusp at the boundary of AdS_4 at $z = 0$ is manifestly enforced by the relation $x^+x^- = -\frac{1}{2}z^2$.

3.2.1 Classical level

In AdS_5, the effective string tension reads, in terms of the 't Hooft coupling

$$T = \frac{R_{AdS}^2}{2\pi\alpha'} = \frac{\sqrt{\lambda}}{2\pi} \tag{35}$$

We compute the first term by plugging the solution (34) in the action.

$$L_{E,0} = \dot{z}^2 + \frac{1}{z^4} z'^M z'^M = \frac{1}{4\tau\sigma} \tag{36}$$

$$S_{E,0} = \frac{1}{4} T \int d\tau \int_0^{+\infty} d\sigma \frac{1}{\tau\sigma} = \frac{T}{4} \int dt \int_{-\infty}^{+\infty} ds = \frac{1}{4} T V_2 \tag{37}$$

Hence, at first order, we have

$$f^{(0)}(\lambda) = 2T = \frac{\sqrt{\lambda}}{\pi} \Longrightarrow \boxed{a_0 = \frac{\sqrt{\lambda}}{\pi}} \tag{38}$$

3.2.2 One loop

We now turn on the fluctuations of the coordinates and compute the Lagrangian at order 2 in them. This is formula (3.14) in [5], which appendix A shows how to recover.

The interpretation regarding the spectrum is straightforward for the bosonic part, since the propagator is diagonal: one field $\tilde{\phi}$, $m^2 = 1$, two fields x, x^*, $m^2 = 1/2$, five fields y^a, $m^2 = 0$.

The fermionic part needs more treatment. Using the notation $\Theta = (\theta^i, \theta_i, \eta^i, \eta_i)$ (we drop the tildas for convenience), we have $L_F = i\Theta K_F \Theta^T$, where:

$$K_F = \begin{pmatrix} 0 & ip_0\mathbf{1}_4 & -(ip_1 + \frac{1}{2})\rho^6 & 0 \\ ip_0\mathbf{1}_4 & 0 & 0 & -(ip_1 + \frac{1}{2})\rho_6^\dagger \\ +(ip_1 - \frac{1}{2})\rho^6 & 0 & 0 & ip_0\mathbf{1}_4 \\ 0 & +(ip_1 - \frac{1}{2})\rho_6^\dagger & ip_0\mathbf{1}_4 & 0 \end{pmatrix} \tag{39}$$

with $(p_0, p_1) = -i(\partial_t, \partial_s)$.

The propagator can be put in a block diagonal form and the computation of the determinant yields

$$\det(K_F) = \left(p^2 + \frac{1}{4}\right)^8 \qquad p^2 = p_0^2 + p_1^2 \tag{40}$$

The fermionic spectrum has therefore eight fields of mass $m^2 = 1/4$.

We can conclude the calculation:

$$W_1 = V_2 \frac{1}{2} \int\int \frac{d^2p}{(2\pi)^2} \left[\ln(p^2+1) + 2\ln(p^2 + \frac{1}{2}) + 5\ln(p^2) - 8\ln(p^2 + \frac{1}{4})\right] \tag{41}$$

The computation of this last integral yields

$$W_1 = -\frac{3\ln(2)}{8\pi} V_2 \Longrightarrow \boxed{a_1 = -3\ln(2)} \tag{42}$$

3.2.3 Two loops

The expansion of the Lagrangian at quartic order is not really difficult and is reproduced in [5], paragraph 4. Let's for the moment have a look at the general approach.

After expanding the Lagrangian, we can compute the vertex factor by looking at the interaction terms. For example, the term

$$S^{(3)}_{\tilde{\phi}\tilde{x}\tilde{x}^*} = -2\int dtds\tilde{\phi}\left|\partial_s\tilde{x} - \frac{1}{2}\tilde{x}\right|^2 \tag{43}$$

means that when writing a vertex between the fields ϕ, x and x^*, we have to write a $-2(p_1^2 + \frac{1}{4})$ contribution, where p_1 is the spatial momentum of x. The propagators are computed from the inverse of the quadratic lagrangian.

The contributing diagrams can also be read from the lagrangian. In our case, they only are the sunset diagram (requiring two cubic vertices), the double-bubble (one quartic vertex) and the double tadpole.

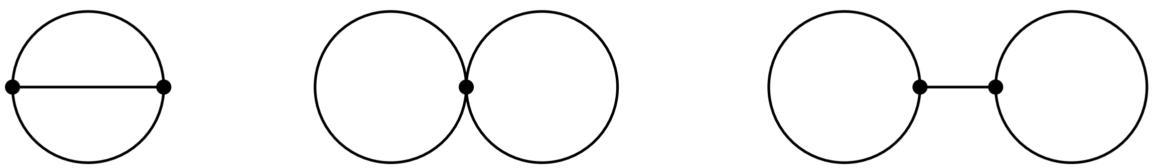

Figure 3: Contributing diagrams in $AdS_5 \times S^5$, which are the same in $AdS_4 \times \mathbb{CP}^3$: sunset, double bubble, double tadpole

We use the following formula for the effective action (which is a low-order expansion of the $Z = \exp(-W)$ relation)

$$W^{(2)} = -\ln\ Z = \langle S_{int}\rangle - \frac{\langle S_{int}^2\rangle}{2} \tag{44}$$

The first term will use quartic interactions, the second cubic ones.

We will not compute all the terms (see [3]), but we give an example of a bosonic computation. First note that all results can be given as combinations of the following integrals

$$I[m^2] = \int d^2p \frac{1}{p^2+m^2} \tag{45}$$

$$I[m_1^2, m_2^2, m_3^2] = \int d^2p\ d^2q\ d^2r \frac{\delta^{(2)}(p+q+r)}{(p^2+m_1^2)(q^2+m_2^2)(r^2+m_3^2)} \tag{46}$$

The first term is UV divergent, and the second one behaves properly whenever $m_i \neq 0$. In order for the final result to be finite, it will have to be checked that all divergent contributions cancel.

Consider the term given as an example. It gives rise to a contribution to the effective action as

$$W_{2,\phi xx^*} = -\frac{1}{2}\langle S^3_{\phi xx} S^3_{\phi xx}\rangle \tag{47}$$

We have two vertices and three propagators, thus our computation yields, in momentum space

$$W_{2,\phi xx^*} = -\frac{1}{2}\int d^2p d^2q d^2r \delta^{(2)}(p+q+r)\frac{(1+4q_1^2)(1+4r_1^2)}{(p^2+1)(q^2+\frac{1}{2})(r^2+\frac{1}{2})} \tag{48}$$

The explicit computation of this integral requires the Passarino-Veltman reduction for scalar integrals[16]. We finally obtain

$$W_{2,\phi xx^*} = \frac{1}{4} I[1,\frac{1}{2},\frac{1}{2}] \tag{49}$$

All the bosonic diagrams can be treated in such a way. For the fermionic ones, the same framework can be used, but one should be cautious to define an order in the diagram, since the fermions anticommute. Besides, at least in [5], the propagators include Dirac matrices, so the trace must be taken in order to get a gauge-invariant quantity.

After summing all the contributions at two-loops, we get a finite result, namely

$$\boxed{a_2 = -K} \qquad K = \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^2} \quad \text{Catalan's constant} \tag{50}$$

3.3 Computation in $AdS_4 \times \mathbb{CP}^3$

We are now in position to do the same computation in the framework of $AdS_4 \times CFT_3$, using namely as background $AdS_4 \times \mathbb{CP}^3$. Surprisingly, the most of the toolkit used in the previous case (gauge, cusp, coordinates, ...) can be translated in this space. We will compute the one and two loop anomalous dimension for the cusped Wilson loop. We use as basis the Lagrangian found in [6], expression (7), (8), which is obtained from double dimensional reduction of the $OSp(4|8)/(SO(1,3)\times SO(7))$ supercoset membrane action on the $D = 11$ maximally supersymmetric background $AdS_4 \times S^7$ in [21].

The coordinates are:

$$AdS_4 : x^{\pm} = x^2 \pm x^0, x^1, \varphi \qquad \mathbb{CP}^3 : z^M,\ M = 1\ldots 6 \equiv (z^a, \bar{z}_a),\ a = 1,2,3 \tag{51}$$

with (x^0, x^1, x^2) the three-dimensional Minkowski coordinates. The radial coordinate is defined as $z = e^{2\varphi}$. The metric associated is

$$ds^2_{AdS} = \frac{e^{-4\varphi}}{4}(-\mathrm{d}x_0^2 + \mathrm{d}x_1^2 + \mathrm{d}x_2^2 + (\mathrm{d}(e^{2\varphi}))^2) = \frac{e^{-4\varphi}}{4}(\mathrm{d}x^+\mathrm{d}x^- + (\mathrm{d}x^1)^2) + \mathrm{d}\varphi^2 \tag{52}$$

$$ds^2_{\mathbb{CP}^3} = g_{MN}\mathrm{d}z^M\mathrm{d}z^N = g_{ab}\mathrm{d}z^a\mathrm{d}z^b + g^{ab}\mathrm{d}\bar{z}_a\mathrm{d}\bar{z}_b + 2g^a{}_b\mathrm{d}\bar{z}_a\mathrm{d}z^b \tag{53}$$

[16] We refer to any good textbook on QFT for this technique

There's an important subtlety here: contrary to $AdS_5 \times S^5$, $AdS_4 \times \mathbb{CP}^3$ is not maximally supersymmetric[17]. Therefore, the AdS radius receives quantum corrections. The derivation of these corrections can be found in [15] and yields to the following expression for the string tension (in the 't Hooft limit)

$$T = \sqrt{\frac{1}{2}\left(\lambda - \frac{1}{24}\right)} \tag{54}$$

It is important to emphasize that the string perturbative expansion is an expansion in inverse string tension, whose coefficients are not affected by the corrections (57). Namely, we can rewrite (27) more generally as

$$f(g) = g\left(a_0 + \frac{a_1}{g} + \frac{a_2}{g^2} + \dots\right) \tag{55}$$

The shift plays a role when expressing the formula above in terms of the 't Hooft coupling, and one finds

$$f(\lambda) = \sqrt{2\lambda}\left(1 + \frac{a_1}{\sqrt{\lambda}} + \frac{a_2 - \frac{1}{48}}{\lambda} + \dots\right), \tag{56}$$

where we have already substituted the leading classical result [16].

Therefore, the shift modifies the result only at two-loop order and beyond. We can redefine $a_2 \equiv a_2 - \frac{1}{48}$.

3.3.1 ABJM interpolating $h(\lambda)$ function

Of course, the result of our direct string computation should be eventually compared with the prediction that on the gauge side can be got via integrability, a feature which the AdS_4/CFT_3 system seems to share [22] with the AdS_5/CFT_4 case [23]. But there's more. In [1], Gromov and Vieira made a conjecture which allows the computation of the cusp anomalous dimension in ABJM theory from the one computed in $\mathcal{N}=4$ SYM via Bethe equations. The details of this paper are involved, but the main idea stands in the replacement

$$f_{CS}(\lambda) = \frac{1}{2} f_{\mathcal{N}=4}(\lambda)\Big|_{\frac{\sqrt{\lambda}}{4\pi} \to h(\lambda)} \tag{57}$$

Above, $h(\lambda)$ is a non-trivial function of the coupling which appears in the "magnon" dispersion relation that is at the basis of all calculations based on integrability. Its exact form is unknown by first principles, but an all loop conjecture has been made in [2], based on a comparison with localization results, of which the weak and string coupling expansions are

$$\begin{cases} h(\lambda) \underset{\lambda<<1}{=} \lambda - \frac{\pi^2}{3}\lambda^3 + \frac{5\pi^4}{12}\lambda^5 - \frac{893\pi^6}{1260}\lambda^7 + \mathcal{O}(\lambda^9) \\ h(\lambda) \underset{\lambda>>1}{=} \sqrt{\frac{1}{2}\left(\lambda - \frac{1}{24}\right)} - \frac{\ln(2)}{2\pi} + \mathcal{O}(e^{-\pi\sqrt{8\lambda}}) \end{cases} \tag{58}$$

If we use this for large coupling (the case which will be of interest for us) and plug it in the $\mathcal{N}=4$ SYM expression of $f(\lambda)$, we get:

[17] Only 24 of the 32 supersymmetries are left.

$$f_{CS}(\lambda) = \sqrt{2\lambda}\left[1 - \frac{5\ln(2)}{2\pi\sqrt{2}\sqrt{\lambda}} - \left(\frac{1}{48} + \frac{K}{8\pi^2}\right)\frac{1}{\lambda}\right] \tag{59}$$

That is

$$a_1 = \frac{-5\ln(2)}{2\pi\sqrt{2}} \quad a_2 = -\left(\frac{1}{48} + \frac{K}{8\pi^2}\right) \tag{60}$$

In the next sections we are going to see whether our direct calculation matches this prediction.

3.3.2 Lagrangian expansion

Starting from the same background solution (34) as in $AdS_5 \times S^5$, we consider the action of [6] which we report in appendix B, and expand the fields in small fluctuations in order to compute the cusp anomalous dimension at one and two loop order in σ-model perturbation theory.

Fluctuations

We define the fluctuations. The expansion of $\mathbb{CP}^3$ metric, hatted variables and spin connection can be found in appendix B.

- *AdS_4 bosonic sector*:

$$\tilde{z} = ze^{2\tilde{\varphi}} \Rightarrow \varphi = \frac{1}{4}\ln\left(\frac{\tau}{\sigma}\right) + \tilde{\varphi}$$

$$\tilde{x^1} = 2\sqrt{\frac{\tau}{\sigma}}\tilde{x}^1$$

- *$\mathbb{CP}^3$ bosonic sector* :

  We have 6 variables $(z^a, \bar{z}_a), a = 1, 2, 3$.

  We define the fluctuations to simply be

$$\tilde{z^a} \equiv z^a \qquad \tilde{\bar{z}}_a \equiv \bar{z}_a$$

- *Fermions:*

  We first rotate the fermions in order to collect the same powers of e^{φ} in the expressions :

$$\theta_a \to \theta_a \qquad \theta_4 \to e^{-\varphi}\theta_4 \qquad \eta_a \to e^{-2\varphi}\eta_a \qquad \eta_4 \to e^{-\varphi}\eta_4$$

  and same thing for the conjugate variables.

  nota: When considering derivatives, we can forget about the presence of terms involving derivatives of φ. It can be checked out that they cancel.

Then, we define the fluctuations

$$\text{fermion} = \sqrt{\frac{2}{\sigma}}\widetilde{\text{fermion}}$$

nota: It can be checked out that this normalization has only one purpose: combined with a further redefinition of the worldsheet coordinates $t = \log\tau$ and $s = \log\sigma$, it is such that the coefficients of the fluctuations become constant, namely (τ, σ)-independent.

One loop

After plugging in the fluctuations, the bosonic Lagrangian in AdS_4 reads, at quadratic order:

$$\mathcal{L}_B = -\frac{T}{2}\left\{(\partial_t x^1)^2 + (\partial_s x^1)^2 + \frac{1}{2}\tilde{x}^2 + (\partial_t\varphi)^2 + (\partial_s\varphi)^2 + \varphi^2\right\} \tag{61}$$

Hence, the bosonic spectrum in AdS_4 consists in:

1. a massive field x^1 with mass $m^2 = 1/2$.
2. a massive field φ with mass $m^2 = 1$.

Regarding $\mathbb{CP}^3$, after expanding, we obtain

$$\mathcal{L}_{\mathbb{CP}^3} = -\frac{T}{2}\left\{\partial_t z^a \partial_t \bar{z}_a + \partial_s z^a \partial_s \bar{z}_a\right\} \tag{62}$$

We find that we have 6 massless fields on $\mathbb{CP}^3$.

Let us finally consider the fermions. After rotating the fermions (appendix) and plugging in the fluctuations, we get the propagator K_F defined by $\mathcal{L}_{\mathcal{F}} = i^t\Theta K_F \Theta, \Theta = (\theta_i, \theta_4, \bar{\theta}^i, \bar{\theta}^4, \eta_i, \eta_4, \bar{\eta}^i, \bar{\eta}^4)$:

$$K_F = \begin{pmatrix} 0 & 0 & ip_0 & 0 & 0 & 0 & ip_1 - \frac{1}{2} & 0 \\ 0 & 0 & 0 & ip_0 & 0 & 0 & 0 & ip_1 \\ ip_0 & 0 & 0 & 0 & -ip_1 + \frac{1}{2} & 0 & 0 & 0 \\ 0 & ip_0 & 0 & 0 & 0 & -ip_1 & 0 & 0 \\ 0 & 0 & -ip_1 - \frac{1}{2} & 0 & 0 & 0 & ip_0 & 0 \\ 0 & 0 & 0 & -ip_1 & 0 & 0 & 0 & ip_0 \\ ip_1 + \frac{1}{2} & 0 & 0 & 0 & ip_0 & 0 & 0 & 0 \\ 0 & ip_1 & 0 & 0 & 0 & ip_0 & 0 & 0 \end{pmatrix} \tag{63}$$

The computation of its determinant is straightforward, and gives

$$\det K_F = (p^2)^2(p^2 + \frac{1}{4})^6 \tag{64}$$

where $p^2 = p_0^2 + p_1^2$.

Hence, the fermionic spectrum consists in

- two massless fields.
- six massive fields with mass $m^2 = 1/4$.

Given the previous results for the fermionic spectrum, we know from [5] that the one-loop cusp anomaly is given by

$$W_1 = \frac{1}{2}\int \frac{d^2p}{(2\pi)^2}\left\{\ln(p^2+1) + \ln\left(p^2+\frac{1}{2}\right) + 6\ln(p^2) - 2\ln(p^2) - 6\ln\left(p^2+\frac{1}{4}\right)\right\} = -\frac{5\ln 2}{16\pi}\underbrace{\int dtds}_{V_2} \tag{65}$$

Which yields

$$\boxed{a_1 = \frac{8W_1}{\sqrt{2}V} = -\frac{5\ln(2)}{2\pi\sqrt{2}}} \tag{66}$$

Two loops

We now go to cubic/quartic order in fluctuations. The derivation is quite lengthy, and the result can be found in Appendix C.

Then, we use the same procedure as for AdS_5: we compute the Feynman diagrams using the expansion to express the vertices, and the inverse of the quadratic expansion to get the propagators.

All of these diagrams give scalar integrals that must be reduced.

Since we know our final result should be finite, a first approach consisted in checking what the finite contributions give as a result. After some work, we got the results summarized in our paper [3]

W_2	Boson	Fermion
Sunset	$\frac{V_2}{\sqrt{2\lambda}}\left(\frac{1}{2}I[1,\frac{1}{2},\frac{1}{2}] + 2I[1]^2\right)$	$\frac{V_2}{\sqrt{2\lambda}}\left(-\frac{3}{8}I[\frac{1}{2},\frac{1}{4},\frac{1}{4}] + 6I[\frac{1}{4}]I[1]\right)$
Double-bubble/tadpole	$\frac{V_2}{\sqrt{2\lambda}}\left(-2I[1]^2\right)$	$\frac{V_2}{\sqrt{2\lambda}}\left(-6I[\frac{1}{4}]I[1]\right)$

from which we get

$$W_2 = \frac{V_2}{\sqrt{2\lambda}}\left(\frac{1}{2}I[1,\frac{1}{2},\frac{1}{2}] - \frac{3}{8}I[\frac{1}{2},\frac{1}{4},\frac{1}{4}]\right) = -\frac{K}{32\pi^2}\frac{V_2}{\sqrt{2\lambda}} \tag{67}$$

and

$$a_2 = -\frac{K}{4\pi^2} \Rightarrow \boxed{a_2 = -\frac{K}{4\pi^2} - \frac{1}{48}} \tag{68}$$

Afterwards, though, an important work was devoted to checking that the divergences (caused by terms of the type $I[m^2]$ or $I[m_1^2, m_2^2, m_3^2]$, where at least one of the three masses is zero) effectively cancel one another. This is indeed the case, which is a remarkable proof of the consistency of the action of [6] in the $AdS_4 \times \mathbb{CP}^3$ background, at least up to second order in σ-model perturbation theory.

3.4 Results

The results (66) and (68) of our direct sigma-model calculation in the $AdS_4 \times \mathbb{CP}^3$ background and the prediction (60) coming from integrability (with the assumption of $h(\lambda)$ given by the conjecture of [2]) agree perfectly. This can be taken as evidence, albeit indirect, of quantum integrability for the Type IIA $AdS_4 \times CP^3$ superstring in this gauge. On the other hand, we could interpret this result as a non-trivial consistency check of several conjectures, like the one of [2] for $h(\lambda)$, the supposed integrability of strings in this background, the radius shift of [15] and the AdS/CFT correspondence itself.

Conclusion

In this report, I presented the work achieved during the five months of my internship. A long part was devoted to understanding the foundations of string theory (briefly summarized in section 1). This report was limited to describing string actions, their symmetries, and the introduction of supersymmetry.

Then, I got familiar with the AdS/CFT correspondence and its implications, understanding how gauge and string sides are linked. In my internship, I was concerned with the AdS_5/CFT_4 and AdS_4/CFT_3 cases.

Finally, we have computed the one- and two-loop cusp anomalous dimension in both cases. The first case has been explored in [5], and redoing the calculation in it was merely a warm-up exercise. The second case is an original result, appeared in [3]. We proved that up to second order, the ansatz in [1] can be trusted to extrapolate the AdS_5 results in the AdS_4 context, thanks to which more and more light can be shed on the links between AdS_5 and AdS_4. Also, the mutual consistency of several ingredients - our direct perturbative string calculation, the corrected dictionary of [15], the prediction (60) from the Bethe Ansatz [1] and the conjecture of [2] for the interpolating function $h(\lambda)$ - provides highly non-trivial evidence in support of the proposal [2] for the interpolating function $h(\lambda)$ of ABJM theory, and furnishes an indirect check of the quantum integrability of the $AdS_4 \times \mathbb{CP}^3$ superstring theory in this κ-symmetry light-cone gauge.

Appendices

A Quadratic expansion of $AdS_5 \times S^5$

Let's establish the expression of the Lagrangian at quadratic order $\mathcal{L}_2$. We start from the expression (3.9) and expand every term at quadratic order using (3.4), (3.5), (3.6). Following the paper, we explicitely discard every total derivative term. The equal sign must therefore be understood as "equal up to a total derivative".

1. $$\left|\partial_t \tilde{x} + \frac{1}{2}\tilde{x}\right|^2 = |\partial_t \tilde{x}|^2 + \frac{1}{4}\tilde{x}^2$$

2. $$\frac{1}{\tilde{z}^4}\left|\partial_s \tilde{x} - \frac{1}{2}\tilde{x}\right|^2 = |\partial_s \tilde{x}|^2 + \frac{1}{4}\tilde{x}^2$$

3. $$\left(\partial_t \tilde{z}^M + \frac{1}{2}\tilde{z}^M + \frac{i}{\tilde{z}^2}\tilde{\eta}^i (\rho^{MN})^i{}_j \tilde{\eta}^j \tilde{z}^N\right)^2$$

   The fermionic part is already quadratic, hence we take it into account.

   $$= (\partial_t \tilde{z}^M)^2 + \frac{1}{4}(\tilde{z}^M)^2 = (\partial_t \tilde{\phi})^2 + (\partial_t y^a)^2 + \frac{1}{2}\tilde{\phi}^2$$

4. $$\frac{1}{\tilde{z}^4}(\partial_s \tilde{z}^M - \frac{1}{2}\tilde{z}^M)^2 = (\partial_s \tilde{z}^M)^2 + \frac{1}{4}(\tilde{z}^M)^2 = (\partial_s \tilde{\phi})^2 + (\partial_s y^a)^2 + \frac{1}{2}\tilde{\phi}^2$$

5. $$i(\tilde{\theta}^i \partial_t \tilde{\theta}_i + \tilde{\eta}^i \partial_t \tilde{\eta}_i + \tilde{\theta}_i \partial_t \tilde{\theta}^i + \tilde{\eta}_i \partial_t \tilde{\eta}^i) = 2i(\tilde{\theta}^i \partial_t \tilde{\theta}_i + \tilde{\eta}^i \partial_t \tilde{\eta}_i)$$

6. The last two terms are easy: to get onl quadratic contributions, we need $M = 6$, yielding to

   $$2i\left[\tilde{\eta}^i (\rho^6)_{ij}(\partial_s \tilde{\theta}^j - \frac{1}{2}\tilde{\theta}^j)\right] + 2i\left[\tilde{\eta}_i ((\rho^6)^\dagger)^{ij}(\partial_s \tilde{\theta}_j - \frac{1}{2}\tilde{\theta}_j)\right]$$

Collecting all the terms, we get indeed the expression (3.14) of $\mathcal{L}_2$.

B Lagrangian in $AdS_4 \times \mathbb{CP}^3$

The κ-symmetry light-cone gauge-fixed Lagrangian of [21] can be written as follows

$$
\begin{aligned}
S &= -\frac{T}{2}\int d\tau\, d\sigma\, L \qquad (69)\\
L &= \gamma^{ij}\Big\{\frac{e^{-4\varphi}}{4}\left(\partial_i x^+\partial_j x^- + \partial_i x^1\partial_j x^1\right) + \partial_i\varphi\partial_j\varphi + g_{MN}\partial_i z^M\partial_j z^N\\
&\quad + e^{-4\varphi}\left[\partial_i x^+\varpi_j + \partial_i x^+\partial_j z^M h_M + e^{-4\varphi}B\partial_i x^+\partial_j x^+\right]\Big\}\\
&\quad - 2\,\varepsilon^{ij}e^{-4\varphi}\left(\omega_i\partial_j x^+ + e^{-2\varphi}C\partial_i x^1\partial_j x^+ + \partial_i x^+\partial_j z^M\ell_M\right)
\end{aligned}
$$

where the string tension T has been defined in (54) and the following quantities

$$\varpi_i = i\left(\partial_i\theta_a\bar\theta^a - \theta_a\partial_i\bar\theta^a + \partial_i\theta_4\bar\theta^4 - \theta_4\partial_i\bar\theta^4 + \partial_i\eta_a\bar\eta^a - \eta_a\partial_i\bar\eta^a + \partial_i\eta_4\bar\eta^4 - \eta_4\partial_i\bar\eta^4\right) \qquad (70)$$

$$\omega_i = \hat\eta_a\hat\partial_i\bar\theta^a + \hat\partial_i\theta_a\hat{\bar\eta}^a + \frac{1}{2}\left(\partial_i\theta_4\bar\eta^4 - \partial_i\eta_4\bar\theta^4 + \eta_4\partial_i\bar\theta^4 - \theta_4\partial_i\bar\eta^4\right) \qquad (71)$$

$$B = 8\,\left[(\hat\eta_a\hat{\bar\eta}^a)^2 + \varepsilon_{abc}\hat{\bar\eta}^a\hat{\bar\eta}^b\hat{\bar\eta}^c\bar\eta^4 + \varepsilon^{abc}\hat\eta_a\hat\eta_b\hat\eta_c\eta_4 + 2\eta_4\bar\eta^4\left(\hat\eta_a\hat{\bar\eta}^a - \theta_4\bar\theta^4\right)\right] \qquad (72)$$

$$C = 2\,\hat\eta_a\hat{\bar\eta}^a + \theta_4\bar\theta^4 + \eta_4\bar\eta^4 \qquad (73)$$

$$h_M = 2\,\left[\Omega^a_M\varepsilon_{abc}\hat{\bar\eta}^b\hat{\bar\eta}^c - \Omega_{aM}\varepsilon^{abc}\hat\eta_b\hat\eta_c + 2\left(\Omega_{aM}\hat{\bar\eta}^a\bar\eta^4 - \Omega^a_M\hat\eta_a\eta_4\right) + 2\left(\theta_4\bar\theta^4 + \eta_4\bar\eta^4\right)\tilde\Omega_a{}^a{}_M\right] \qquad (74)$$

$$\ell_M = 2\,i\,\left[\Omega_{aM}\hat{\bar\eta}^a\bar\theta^4 + \Omega^a_M\hat\eta_a\theta_4 + \left(\theta_4\bar\eta^4 - \eta_4\bar\theta^4\right)\tilde\Omega_a{}^a{}_M\right] \qquad (75)$$

include fermions up to the fourth power.

Above, the fermionic coordinates η_a and θ_a (and their conjugates) transform in the fundamental (antifundamental) representation of $SU(3)$ ($a = 1, 2, 3$), and correspond to the unbroken 24 supersymmetries of the $AdS_4 \times \mathbb{CP}^3$ background. The remaining fermions η_4, θ_4 and their conjugates originate from the eight broken supersymmetries. A manifest symmetry of the action is thus the $SU(3)$ subgroup of the $SU(4)$ global symmetry of $\mathbb{CP}^3$. As in the $AdS_5 \times S^5$ case [11] the action is quadratic in the θ-fermions and quartic in the η-fermions. The Ω^a_M and Ω_{aM} appearing in the Lagrangian are the complex vielbein of $\mathbb{CP}^3$, $ds^2_{\mathbb{CP}^3} = \Omega^a_M\Omega_{aN}\,dz^M\,dz^N$, namely components of the Cartan one-forms of $SU(4)/U(3)$, $\Omega^a = \Omega^a_M\,dz^M$ and $\Omega_a = \Omega_{aM}\,dz^M$. In the construction of [21], $\tilde\Omega_a{}^a$ is associated to a one-form corresponding to the fiber direction of S^7. Its expression is given explicitly below in terms of the $\mathbb{CP}^3$ coordinates. The Ω^a_M and $\tilde\Omega_a{}^a$ appear in [21] in a "dressed" $OSp(6|4)/(SO(1,3)\times U(3))$ supercoset element where the dressing incorporates the information on the broken supersymmetries and $U(1)$ fiber direction. In (70), hatted quantities are related to un-hatted ones via a rotation by matrices T (similar matrices were conveniently introduced in [11]) which depend on the $\mathbb{CP}^3$ coordinates and act as follows on e.g. a η_a fermion

$$\hat\eta_a = T_a{}^b\,\eta_b + T_{ab}\,\bar\eta^b \qquad\qquad \hat{\bar\eta}^a = T^a{}_b\,\bar\eta^b + T^{ab}\,\eta_b\,. \qquad (76)$$

The T matrix element can be expanded at fourth order

- $T_{ab} = i\varepsilon_{acb}z^c(1 - \frac{|z|^2}{6})$

- $T^{ab} = -i\varepsilon^{acb}\bar{z}_c(1 - \frac{|z|^2}{6})$
- $T_a{}^b = T^b{}_a = (1 - \frac{|z|^2}{2} + \frac{|z|^4}{24})\delta^a_b + \frac{1}{2}\bar{z}^a z_b(1 - \frac{|z|^2}{12})$

For the worldsheet metric, Weyl invariance allows us to choose, in Euclidean space:

$$\gamma^{ij} = \begin{pmatrix} z^2 & 0 \\ 0 & z^{-2} \end{pmatrix}$$

The metric on $\mathbb{CP}^3$ is defined as follows

$$ds^2 = g_{ab}dz^a dz^b + g^{ab}d\bar{z}_a d\bar{z}_b + 2g_a{}^b dz^a d\bar{z}_b$$

Nota: we will quasi-always trade the $M \in \{1, \ldots, 6\}$ indices with $a \in \{1, 2, 3\}$ indices through the substitution $z^M \to z^a, \bar{z}_a$

Taylor expansion of the metric yields at fourth order

- $g_{ab} = (\frac{1}{3} - \frac{8}{45}|z|^2)\bar{z}_a\bar{z}_b$
- $g^{ab} = (\frac{1}{3} - \frac{8}{45}|z|^2)z^a z^b$
- $g_a{}^b = (\frac{1}{2} - \frac{|z|^2}{6} + \frac{|z|^4}{45})\delta^b_a - \bar{z}_a z^b(\frac{1}{6} - \frac{7}{45}|z|^2)$

Finally, the spin connection can also be Taylor expanded. Pay attention to the fact that (41) in [6] omitts one term due to convenience.

- $\Omega_{ab} = \bar{z}_a\bar{z}_b(\frac{1}{3} - \frac{|z|^2}{15})$
- $\Omega^{ab} = z^a z^b(\frac{1}{3} - \frac{|z|^2}{15})$
- $\Omega_a{}^b = \Omega^b{}_a = (1 - \frac{|z|^2}{6} + \frac{|z|^4}{120})\delta^b_a - \bar{z}_a z^b(\frac{1}{6} - \frac{7}{120}|z|^2$
- $\tilde{\Omega}_a{}^a{}_b = i\bar{z}_b(1 - \frac{|z|^2}{3})$
- $\tilde{\Omega}_a{}^{ab} = -iz^b(1 - \frac{|z|^2}{3})$

C Expansion of the $AdS_4 \times \mathbb{CP}^3$ Lagrangian at quartic order

In this section, we give the final result of our expansion at quartic order of the Lagrangian found in [5]. The computation was rather long, hence we do not explicit the calculation.

$$
\begin{aligned}
\mathcal{L}_{(3)} = \;& -8\varphi(\partial_s x^1)^2 - 2\varphi(x^1)^2 + 8\varphi x^1(\partial_s x) + 4\varphi[(\partial_t\varphi)^2 - (\partial_s\varphi)^2] + 4\varphi(\partial_t z^a\partial_t\bar{z}_a - \partial_s z^a\partial_s\bar{z}_a) \\
& +2\varepsilon_{abc}\partial_t z^a\bar{\eta}^b\bar{\eta}^c - 2\varepsilon^{abc}\partial_t\bar{z}_a\eta_b\eta_c + 4\partial_t\bar{z}_a\bar{\eta}^a\bar{\eta}^4 - 4\partial_t z^a\eta_a\eta_4 \\
& i\left\{\left[2i\varepsilon_{acb}z^c\bar{\eta}^b\partial_s\bar{\theta}^a - i\varepsilon_{acb}z^c\bar{\eta}^b\bar{\theta}^a - 8\varphi\eta_a\partial_s\bar{\theta}^a + 4\varphi\eta_a\bar{\theta}^a - 2i\varepsilon^{adc}\eta_a\left(\partial_s\bar{z}_d\theta_c + \bar{z}_d\partial_s\theta_c - \tfrac{1}{2}\bar{z}_d\theta_c\right)\right] + c.c.\right\} \\
& -4i\varphi(\partial_s\theta_4\bar{\eta}^4 - \partial_s\eta_4\bar{\theta}^4 + \eta_4\partial_s\bar{\theta}^4 - \theta_4\partial_s\bar{\eta}^4) + 8i\eta_a\bar{\eta}^a\partial_s x^1 - 4i\eta_a\bar{\eta}^a x^1 + 4i\theta_4\bar{\theta}^4\partial_s x^1 - 2i\theta_4\bar{\theta}^4 x^1 \\
& +4i\eta_4\bar{\eta}^4\partial_s x^1 - 2i\eta_4\bar{\eta}^4 x^1 + 4\partial_s\bar{z}_a\bar{\eta}^a\bar{\theta}^4 + 4\partial_s z^a\eta_a\theta_4
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_{(4)} = \;& 32\varphi^2(\partial_s x^1)^2 + 8\varphi^2(x^1)^2 - 32\varphi^2 x^1(\partial_s x^1) + \tfrac{4}{3}\varphi^4 + 8\varphi^2(\partial_t\varphi)^2 \\
& +8\varphi^2(\partial_s\varphi)^2 + 8\varphi^2(\partial_t z^a\partial_t\bar{z}_a + \partial_s z^a\partial_s\bar{z}_a) + \tfrac{1}{3}\Big[\bar{z}_a\partial_t z^a\bar{z}_b\partial_t z^b + z^a\partial_t\bar{z}_a z^b\partial_t\bar{z}_b \\
& -z^b\bar{z}_b\partial_t z^a\partial_t\bar{z}_a - \bar{z}_a z^b\partial_t z^a\partial_t\bar{z}_b + \bar{z}_a\partial_s z^a\bar{z}_b\partial_s z^b + z^a\partial_s\bar{z}_a z^b\partial_s\bar{z}_b - z^b\bar{z}_b\partial_s z^a\partial_s\bar{z}_a - \bar{z}_a z^b\partial_s z^a\partial_s\bar{z}_b\Big] \\
& -4i\partial_t\bar{z}_a(z^a\eta_b\bar{\eta}^b + \bar{\eta}^a z^b\eta_b) - 4i\varepsilon^{acb}\partial_t\bar{z}_a\bar{z}_c\eta_b\bar{\eta}^4 - 2i\varepsilon_{acb}\partial_t z^a z^c\bar{\eta}^b\eta_4 + 4i(\theta_4\bar{\theta}^4 + \eta_4\bar{\eta}^4)(\partial_t z^b\bar{z}_b - \partial_t\bar{z}_b z^b) \\
& +8\left[(\eta_a\bar{\eta}^a)^2 + \varepsilon_{abc}\bar{\eta}^a\bar{\eta}^b\bar{\eta}^c\bar{\eta}^4 + \varepsilon^{abc}\eta_a\eta_b\eta_c\eta_4 + 2\eta_4\bar{\eta}^4(\eta_a\bar{\eta}^a - \theta_4\bar{\theta}^4)\right] + i\Big\{+2z^a\bar{z}_a\bar{\eta}_b\partial_s\theta_b - z^a\bar{z}_a\bar{\eta}^b\theta_b \\
& -2\bar{\eta}^a\bar{z}_a z^b\partial_s\theta_b + \bar{\eta}^a\bar{z}_a z^b\theta_b - 8i\varepsilon_{acb}\varphi z^c\bar{\eta}^b\partial_s\bar{\theta}^a + 4i\varepsilon_{acb}\varphi z^c\bar{\eta}^b\bar{\theta}^a + 16\varphi^2\eta_a\partial_s\bar{\theta}^a - 8\varphi^2\eta_a\bar{\theta}^a \\
& -2\eta_a\partial_s\bar{\theta}^a|z|^2 + \eta_a\bar{\theta}^a|z|^2 + 2\eta_a\partial_s\bar{\theta}^c\bar{z}_c z^a - \eta_a\bar{\theta}^c\bar{z}_c z^a + 8i\varphi\eta_a\varepsilon^{acb}\bar{z}_c\partial_s\theta_b - 4i\varphi\eta_a\varepsilon^{acb}\bar{z}_c\theta_b + c.c. \\
& +8\varphi^2(\partial_s\theta_4\bar{\eta}^4 - \partial_s\eta_4\bar{\theta}^4 + \eta_4\partial_s\bar{\theta}^4 - \theta_4\partial_s\bar{\eta}^4) + 8i\varepsilon_{acb}z^c\bar{\eta}^b\bar{\eta}^a\partial_s x^1 - 4i\varepsilon_{acb}z^c\bar{\eta}^b\bar{\eta}^a x^1 \\
& -8i\varepsilon^{adc}\eta_a\bar{z}_d\eta_c\partial_s x^1 + 4i\varepsilon^{adc}\eta_a\bar{z}_d\eta_c x^1 - 48\varphi\eta_a\bar{\eta}^a\partial_s x^1 + 24\varphi\eta_a\bar{\eta}^a x^1 - 24\varphi\theta_4\bar{\theta}^4\partial_s x^1 \\
& +12\varphi\theta_4\bar{\theta}^4 x^1 - 24\varphi\eta_4\bar{\eta}^4\partial_s x^1 + 12\varphi\eta_4\bar{\eta}^4 x^1 - 4\varepsilon^{acb}\partial_s\bar{z}_a\bar{z}_c\eta_b\bar{\theta}^4 - 4\varepsilon_{acb}\partial_s z^a z^c\bar{\eta}^b\theta^4 \\
& +16i\varphi\partial_s\bar{z}_a\bar{\eta}^a\bar{\theta}^4 + 16i\varphi\partial_s z^a\eta_a\theta_4 + 4\left[\theta_4\bar{\eta}^4\partial_s z^b\bar{z}_b - \theta_4\bar{\eta}^4\partial_s\bar{z}_b z^b - \eta_4\bar{\theta}^4\partial_s z^b\bar{z}_b + \eta_4\bar{\theta}^4\partial_s\bar{z}_b z^b\right]\Big\}
\end{aligned}
$$

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