From Inflation to Large Scale Structure

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Outline

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Inflation and High Energy Physics

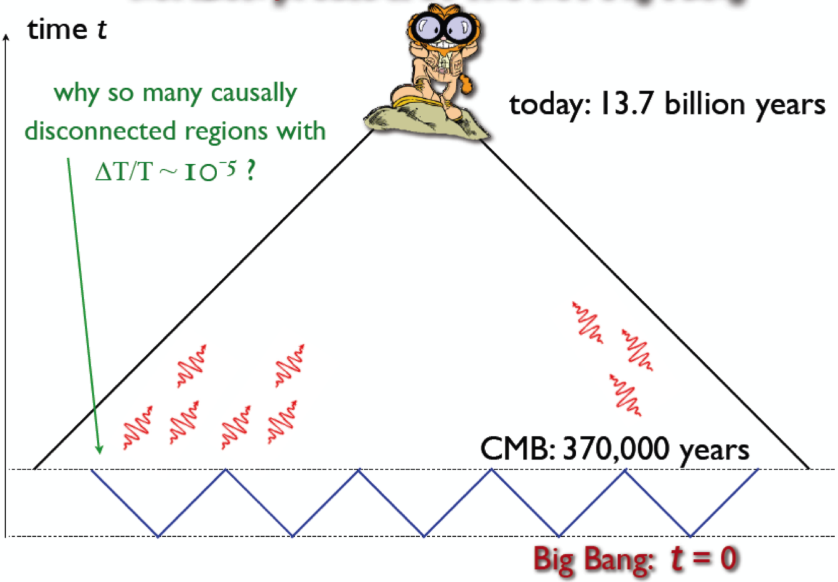
- Inflation and Cosmology after Planck 2015
- Status of Inflationary Models and an Organizing Principle

Cosmological Perturbation Theory and Beyond

- Supernovae Lensing as a Cosmological Probe
- Large Scale Structure and Consistency Relations

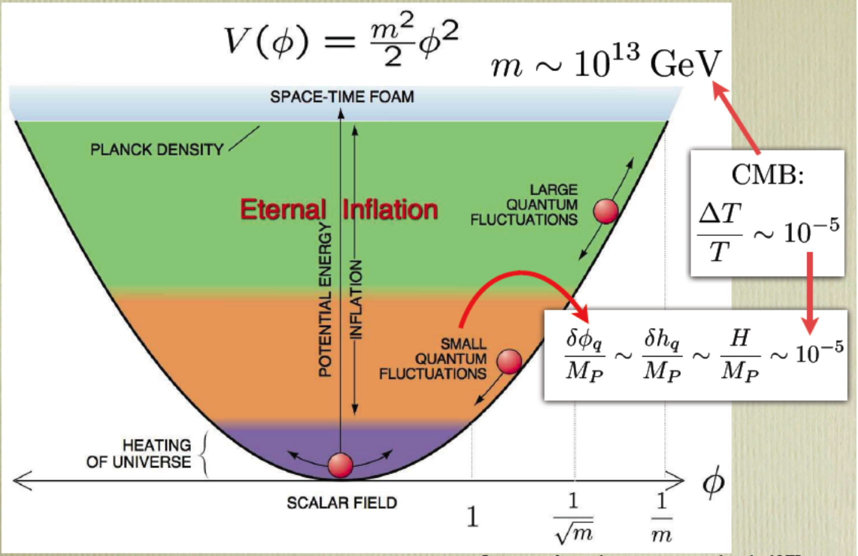
Inflation after Planck 2015

horizon problem of the hot Big Bang

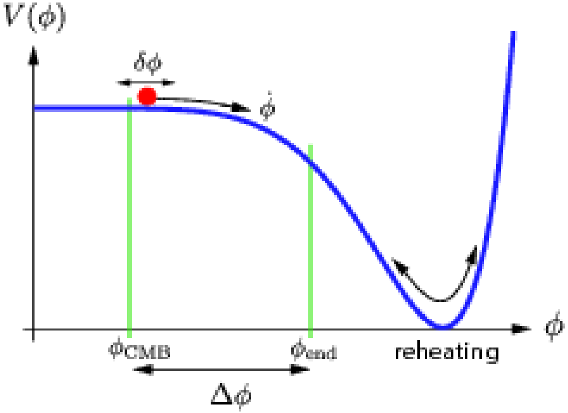


slow-roll inflation ...

[Linde '82]



[picture from lecture notes: Linde '07]

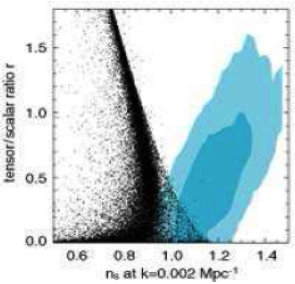


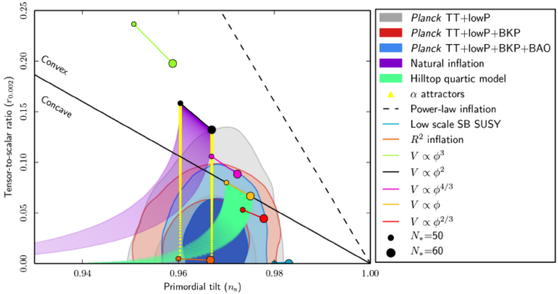
$$\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2 \ll 1,\ \eta = \frac{V''}{V} \ll 1, \cdots\ P_S = A\left(\frac{k}{k_0}\right)^{n_s - 1}$$

$$N_e = \int_{\phi_{CMB}}^{\phi_{END}} \frac{d\phi}{\sqrt{2\epsilon}} \sim 60,\ n_s = 1 + 2\eta - 6\epsilon, \quad r = \frac{P_T}{P_S} = 16\epsilon$$

Inflationary parameters after 12 years of observation

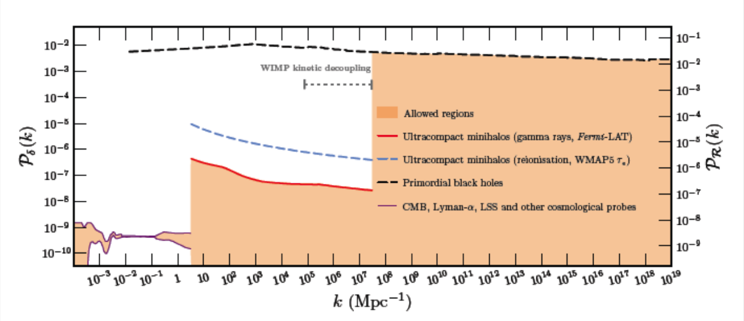
Vanilla monomial models $V = \phi^p$, $p \geq 2$, $r \gtrsim 0.1$ are dead. (SUSY Higgs inflation, IBD & Einhorn 2010)





WMAP 1-year (2003) & PLANCK (2015)

Current Knowledge*



Cosmological Parameters after 12 years of observations

- Basic flat ΛCDM 6 parameters model. $\Omega_{m0}, \Omega_{c0}, H_0, A_s, n_s, \tau$. Except for τ, from 10% accuracy to 1%.

Parameter	TT+lowP 68 % limits	TT+lowP+lensing 68 % limits	TT+lowP+lensing+ext 68 % limits	TT,TE,EE+lowP 68 % limits	TT,TE,EE+lowP+lensing 68 % limits	TT,TE,EE+lowP+lensing+ext 68 % limits
$\Omega_b h^2$	0.02222 ± 0.00023	0.02226 ± 0.00023	0.02227 ± 0.00020	0.02225 ± 0.00016	0.02226 ± 0.00016	0.02230 ± 0.00014
$\Omega_c h^2$	0.1197 ± 0.0022	0.1186 ± 0.0020	0.1184 ± 0.0012	0.1198 ± 0.0015	0.1193 ± 0.0014	0.1188 ± 0.0010
$100\theta_{MC}$	1.04085 ± 0.00047	1.04103 ± 0.00046	1.04106 ± 0.00041	1.04077 ± 0.00032	1.04087 ± 0.00032	1.04093 ± 0.00030
τ	0.078 ± 0.019	0.066 ± 0.016	0.067 ± 0.013	0.079 ± 0.017	0.063 ± 0.014	0.066 ± 0.012
$\ln(10^{10} A_s)$	3.089 ± 0.036	3.062 ± 0.029	3.064 ± 0.024	3.094 ± 0.034	3.059 ± 0.025	3.064 ± 0.023
n_s	0.9655 ± 0.0062	0.9677 ± 0.0060	0.9681 ± 0.0044	0.9645 ± 0.0049	0.9653 ± 0.0048	0.9667 ± 0.0040
H_0	67.31 ± 0.96	67.81 ± 0.92	67.90 ± 0.55	67.27 ± 0.66	67.51 ± 0.64	67.74 ± 0.46
Ω_Λ	0.685 ± 0.013	0.692 ± 0.012	0.6935 ± 0.0072	0.6844 ± 0.0091	0.6879 ± 0.0087	0.6911 ± 0.0062
Ω_m	0.315 ± 0.013	0.308 ± 0.012	0.3065 ± 0.0072	0.3156 ± 0.0091	0.3121 ± 0.0087	0.3089 ± 0.0062

- This is a model dependent statement! Adding parameters increases the allowed values, improves the fit, but with not enough statistical significance.

Status of Inflationary Models and an Organizing Principle

Status of Inflationary models - EFT approach+UV embedding

- Inflation is sensitive to UV physics.
- Organizing Principle: Monge-Ampere eq. $\Leftrightarrow$ Generalized shift symmetry work in progress

$$V(\phi_i) > 0, \quad \det V_{ij} = 0$$

- Large Field Models, $\Delta\phi \gg 1$, $r \sim 0.1 - 0.01$. Shift symmetry, detectable in the near future.
  $V = \Lambda^4(1 - \cos(\phi/f_{eff.}))$, $V \sim \phi \cdots$ (IBD, Pedro & Westphal 2014 x 2)
- 'functional fine-tuning', symmetry is crucial, UV completion is crucial.

Status of Inflationary models - EFT approach+UV embedding

- Inflation is sensitive to UV physics.
- Starobinsky type Models, $\Delta\phi \sim 1$, $r \sim 0.001$. Shift symmetry at $\phi \to \infty$. Could be detected in the near future. (IBD, Jing, Torabian, Westphal, Zarate, 2013).
  $V(\phi \gg 1) \simeq \Lambda^4(1 - e^{-\kappa\phi})^2$.
- 'functional fine-tuning', symmetry is crucial, UV completion is crucial.

Status of Inflationary models - EFT approach+UV embedding

- Inflation is sensitive to UV physics.
- Small Field Models, $\Delta\phi \ll 1$, $r \leq 10^{-4}$, 'Accidental Inflation', theoretically motivated, difficult to detect in the near future. (IBD, Brustein & de-Alwis 2008, IBD & Brustein 2009* (r tunable) , IBD, Jing, Westphal & Wieck 2013) $V \simeq \Lambda^4(1 - a_2\phi^2 + \cdots)$.
- 'parameter fine-tuning' of dimension 6 operators. $\Delta\eta \sim 1$.

Example: Hierarchical Axions

- How to get $f_{eff.} \gg 1$, if $\tilde{f}_i \ll 1$:

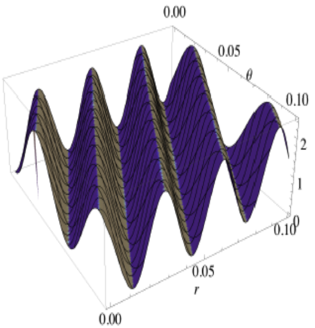
$$V = \Lambda_1^4 \left[1 - \cos\left(\frac{p_1}{\tilde{f}_1}\phi_1 + \frac{p_2}{\tilde{f}_2}\phi_2 \right) \right] + \Lambda_2^4 \left[1 - \cos\left(\frac{q_1}{\tilde{f}_1}\phi_1 + \frac{q_2}{\tilde{f}_2}\phi_2 \right) \right]$$

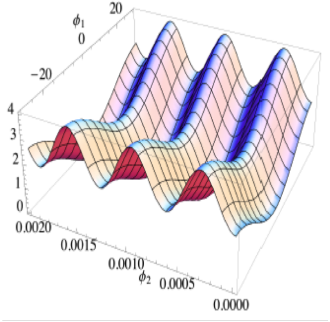
- $V = V(\phi_1 + \phi_2) \Rightarrow \det V_{ij} = 0$. We have a flat direction, corresponding to $\Lambda_1 = 0$ or $p_1 = q_1, p_2 = q_2$.
- The different models now correspond to different breaking patterns. $\Lambda_1 \neq 0, p_2 = 0, p_1 \ll q_1$ is the Hierarchical Axions model. $p_2 = q_2(1 + \delta)$ is the KNP model.

Example: Hierarchical Axions $p_2 = 0$

- $p_2 = 0$. Diagonalizing the mass matrix, and integrating out the heavy axion gives:

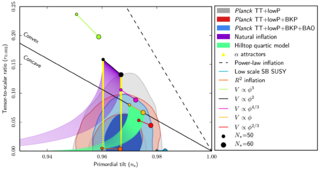
$$f_{eff.} \quad = \quad \tilde{f}_2 \frac{q_1}{q_2 p_1} \gg 1$$





Hierarchical Axions Summary (IBD, Pedro & Westphal 2014 x 2)

- Just 2 axions
- Non-perturbative effects only
- Least amount of tuning of the input parameters.
- The trajectory is contained in a very small field domain. Sheds light on the small vs. large field discussion.
- Combining the model with moduli stabilization in Type IIB string theory.
- Predictions equivalent to 'Natural Inflation' $r \sim 0.05, n_s = 0.96$.



More on String Theory Embedding (IBD, Pedro & Westphal 2014 x 2)

- In string theory we have many moduli that have to be stabilized to avoid decompactification+inflation
- This is achieved by creating a hierarchy between different terms in the lagrangian/potential $V_0 >> V_1 >> V_2$.
- The same hierarchy needed for the original moduli stabilization gives the hierarchy needed for inflation.

What's Next?

- CMB Polarization experiments: CLASS, soon operational $r \leq 0.01$, PIXIE $r \sim 10^{-3}$, PRISM, CoRE+ $\rightarrow$ cosmic variance limited experiment $\sim$ 2030, $r \leq 5 \times 10^{-4}$.
- Measuring the *energy spectrum* of CMB, deviations from black body spectrum, hasn't advanced since COBE, 1992. PIXIE/PRISM (Chluba, Erickcek, IBD 2012)

INFLATION BEYOND CMB!

- Late time measurements: Weak Lensing, Galaxy correlations, SNe, Strong Lensing, BAO and more.
- Challenges:
  1. Designing efficient probes
  2. Accurate theoretical predictions

Outline

Supernovae Lensing as a Cosmological Probe

General Background

- Light-cone averaging method, luminosity-redshift relation $d_L - z$ up to second order in perturbation theory. (IBD, Gasperini, Marozzi, Nugier & Veneziano 2012-2013 x 4)
- Valid for arbitrary geometry, non-perturbative statements.
- UV and IR finite.
- Inhomogeneities are biasing measurements of background quantities, $\frac{\Delta H_0}{H_0} \simeq 2\%$ just from cosmologically induced peculiar velocities! (IBD, Durrer, Marozzi, Schwarz 2014)
- Inhomogeneities serve as probes for Cosmology if we can get rid of systematics.

$$d_L(z) = \frac{1+z}{H_0} \int_0^z \frac{dz'}{\sqrt{\Omega_{m0}(1+z')^3 + 1 - \Omega_{m0}}}$$

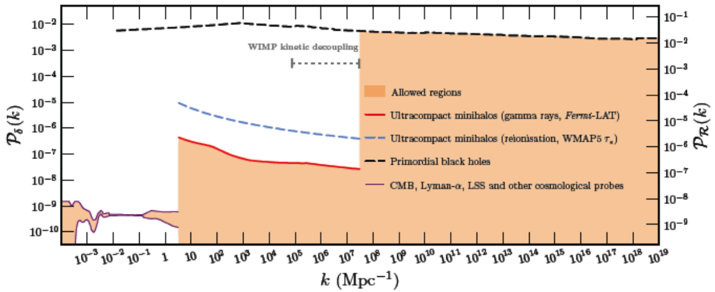
SNIa Lensing (IBD & Kalaydzhyan 2014, IBD 2014)

- Large systematic dispersion of lensing at $z \sim 1$.

$$\sigma_m^2 \simeq \left(\frac{5}{\ln 10}\right)^2 \frac{\pi}{\Delta\tau^2} \int_{\tau_s^{(0)}}^{\tau_o} d\tau_1 \int dk P_\Psi(k,\tau_1) k^2 (\tau_1 - \tau_s^{(0)})^2 (\tau_o - \tau_1)^2$$

- τ conformal time, depends on background parameters only! P_Ψ depends on fluctuations and background parameters. Ψ is the gravitational potential.
- Data: $\sigma_m(z \leq 1) \leq 0.095 (\leq 0.12)$ at $1(2)\sigma$.
- The lensing is sensitive to small scales (quasi) non-linear $k_{NL} > 1 hMpc^{-1}$. We need numerical simulations (IBD & Takahashi TBP) and/or analytical predictions.

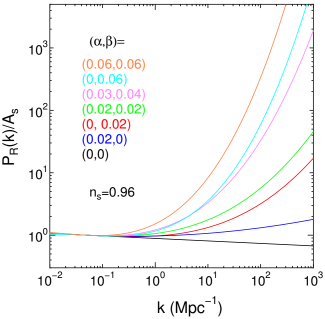
Current Knowledge

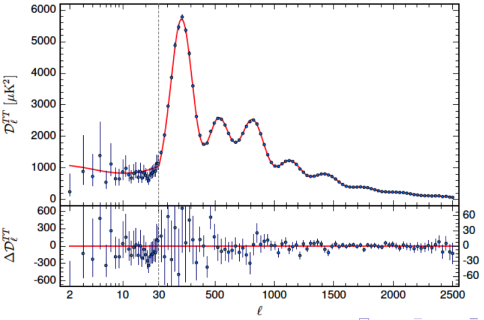


Are we lucky enough to probe Inflation?

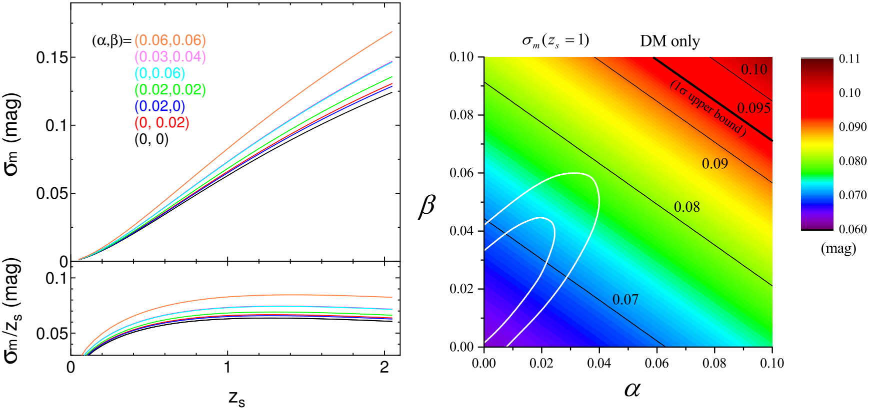
- If yes, $\Rightarrow$ handle on the primordial power spectrum beyond CMB scales!
- $\alpha = 0, \beta = 0.029$ provide the best statistical significant improvement over ΛCDM, $\Delta\chi^2 = 4.9$, to solve the low-l anomaly.

$$\mathcal{P}(k) = A_s \left(\frac{k}{k_0}\right)^{n_s - 1 + (\alpha/2)\ln(k/k_0) + (\beta/6)[\ln(k/k_0)]^2}, \; \alpha \sim \frac{V'''V'}{V^2}, \; \beta \sim \frac{V''''V'^2}{V^3}$$





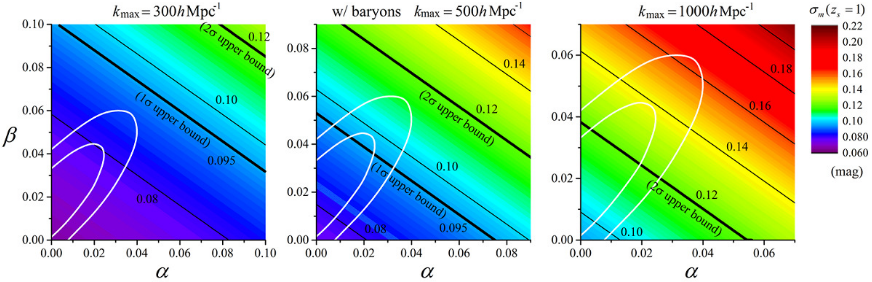
DM only results



Designated numerical simulations ⇒ Memory of initial conditions is largely erased.

Results with baryons

'Competitive' with PLANCK, but UV sensitive!



Lensing Dispersion Summary

- For DM only, the lensing dispersion probes small wave numbers beyond CMB $k \gtrsim 1h\mathrm{Mpc}^{-1}$, not better than PLANCK, because of the erasure of initial conditions by the non-linear evolution.
- The baryons make the fitting formula UV sensitive and swamps initial conditions.This is a problem in weak lensing in general. Better understanding of baryonic processes is key.
- However, SNe do not suffer from 'alignment problems'.
- The upcoming surveys are expected to detect the lensing signal!

Large Scale Structure (LSS) and Consistency Relations (CR)

Consistency Relations

- Relate $(n+1)$- to n-point correlation functions, e.g., bispectrum $\to$ power spectrum
- Analytical result in the squeezed limit $q \to 0$
- Based only on single field inflation and principle of general covariance (unequal time) - Non-perturbative statements.
- Scrutinize two approaches at equal time & angular averaged: (IBD, Konstandin, Porto, Sagunski 2015)
  1. Time flow approach (TF) - transform the fluid eqs. into flow eqs., and apply closure.
  2. Curved background approach (VKPR)- locally the long (soft) mode behaves like background curvature, K.

Consistency Relations

Philosophy: If the approach is of non-perturbative nature, it should definitely fulfill perturbative calculations in the proper limit. For a flat universe (EdS, ΛCDM):

- At linear order: All approaches agree ✓
- generalized to all correlators of δ and $\theta \equiv \boldsymbol{\nabla} \cdot \boldsymbol{v}$ and general cosmologies ✓
- At 1-loop order, for $l \gg k \gg q$: B - bispectrum, P - power spectrum

$$B_{111}^{\text{1-loop}} \ {}_{q\to 0}^{\text{av}} \simeq \left[k^2 \left(\beta + \alpha\, k\, \partial_k\right) P^L(k) \times \int dl\, l^2 \left(\frac{P^L(l)}{l^2}\right) + k^4 \gamma \times \int dl\, l^2 \left(\frac{P^L(l)}{l^2}\right)^2 \right] P^L(q)$$

Consistency Relations

Coefficients α, β, γ at 1-loop order:

Approach	α	β	γ
SPT	$\frac{61}{1890} \simeq 0.032$	$-\frac{3719}{13230} \simeq -0.281$	$\frac{515}{5292} \simeq 0.097$
TF	$-\frac{103}{6930} \simeq 0.015$	$-\frac{233}{1890} \simeq -0.123$	$\frac{271}{19404} \simeq 0.014$
VKPR	$\frac{61}{1890} \simeq 0.032$	$-\frac{3599}{13230} \simeq -0.272$	$\frac{135}{1372} \simeq 0.098$

Deviation of $\mathcal{O}(10^{-2})$ between SPT and VKPR, $\mathcal{O}(1)$ between SPT and TF.

$\Rightarrow$ Both not valid beyond linear order

Why is VKPR ("Guestimating" $\partial_\kappa P_K|_{K=0} = 4/7\partial_\eta P_K|_{K=0}$) so 'successful'?

Curved Background Method

- For spherical perturbations, a long wavelength mode in flat FLRW equals positive curvature:

$$H_K \simeq H\left(1 - \tfrac{1}{3}\delta_L\right), a_K \simeq a\left(1 - \tfrac{1}{3}\delta_L\right),$$
$$K \simeq \tfrac{5}{3}H^2 a^2 \delta_L, \quad \kappa = \tfrac{K}{a^2H^2}$$
$$\bar{\rho}_K = \bar{\rho}(1 + \delta_L)$$

- Fluid equations $\partial_\eta \psi_a = -\Omega_{ab}\,\psi_b + \gamma_{abc}\,\psi_b\psi_c$
  $\psi_a = \{\delta, \boldsymbol{\nabla}\,\boldsymbol{v}\}$
- Judicious change of variables:

$$\tilde{\Omega}_{ab} \simeq \Omega_{ab}(\kappa = 0) + \frac{3\kappa}{14}\begin{pmatrix} 0 & 0 \\ 1 & -1 \end{pmatrix},$$

The new contribution annihilates the growing mode $\psi_a^{(n)} \propto (1,1)$ for all $n \Rightarrow$ Very good accuracy!

Conclusions

Conclusions

CMB and High Energy Physics:

- The inflationary 'landscape' is well understood. The most promising avenue to connect string theory to measurements.
- Polarization and Energy Spectrum measurements $\Rightarrow$ CMB a relevant probe in the next decade and more.

Cosmological Perturbation Theory and Beyond:

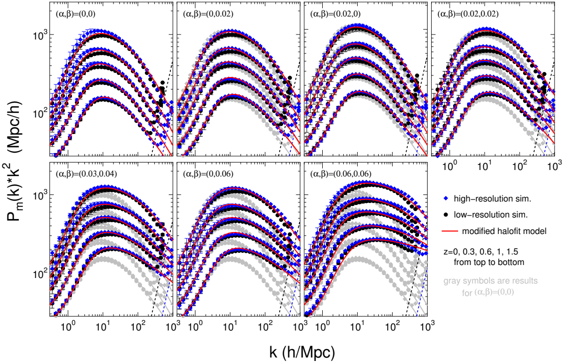
- Lensing of SNIa is useful for cosmology or astrophysics depending on the baryons.
- Combination of late time probes will test extensions of ΛCDM.
- Theoretical understanding of LSS behaviour is emerging as the forefront of theoretical cosmology.

Simulation Results (IBD & Takahashi 1504.xxxxx)

- We ran high and low resolution simulations (DM only) with different values of α, β.

Table : Our Simulation Setting

	$L(h^{-1}\mathrm{Mpc})$	N_{p}^3	$k_{\mathrm{Nyq}}(h\mathrm{Mpc}^{-1})$	$m_{\mathrm{p}}(h^{-1}M_\odot)$	z	N_{r}
high-resolution	100	2048^3	64.3	1.0×10^7	0, 0.3, 0.6, 1, 1.5	3
low-resolution	100	1280^3	40.2	4.1×10^7	0, 0.3, 0.6, 1, 1.5	6



Fluid Equations

- Conservation of mass

  = Continuity equation

$$\frac{\partial \delta}{\partial \tau} + \boldsymbol{\nabla} \cdot \left[(1+\delta) \boldsymbol{v} \right] = 0 \quad (1)$$

- Conservation of momentum

  = Euler equation

$$\frac{\partial v}{\partial \tau} + \mathcal{H} \boldsymbol{v} + (\boldsymbol{v} \cdot \nabla) \boldsymbol{v} = -\boldsymbol{\nabla} \Phi \quad (2)$$

- Poisson equation

$$\triangle \Phi = \frac{3}{2} \Omega_m \mathcal{H}^2 \delta \qquad (3)$$

$\longrightarrow$ highly non-linear differential equations

Physical quantities:

$\longrightarrow$ Density contrast: $\delta = \frac{\rho - \bar{\rho}}{\bar{\rho}}$

$\longrightarrow$ Conformal time τ: $d\tau = \frac{dt}{a}$

$\longrightarrow$ Peculiar velocity $\boldsymbol{v}$

$\longrightarrow$ Expansion rate: $\mathcal{H} = a\, H$

$\longrightarrow$ Gravitational potential Φ (or Ψ)

Standard Perturbation Theory - SPT

- Central quantity: δ
- Assumption: initial density contrast $\delta_0 \ll 1$
- Solution: perturbative expansion in terms of δ_0

$$\delta = \delta^{(1)} + \delta^{(2)} + \ldots \quad \text{with } \delta^{(N)} \propto \delta_0^N$$

- Power spectrum $P(k)$,

  generally P_{ab}, bispectrum B_{abc} etc. $a, b, c = \delta$ or $\theta \equiv \boldsymbol{\nabla} \cdot \boldsymbol{v}$

  = Gaussian average of density contrasts

$$P(k) \propto \langle \delta\delta \rangle$$

$\longrightarrow$ Perturbative (loop) expansion:

$$P(k) \propto \langle \delta^{(1)}\delta^{(1)} \rangle + \ldots = P^{(1)} + \ldots \quad \text{with } P^{(N)} \sim (\delta_0^2)^N$$

Time-flow approach

- Gaussian i.c., solving iteratively with closure $\Rightarrow$ Correlation functions:

$$\begin{aligned} \langle\psi_a\psi_b\rangle &\sim P_{ab}, \\ \langle\psi_a\psi_b\psi_c\rangle &\sim B_{abc}, \\ \langle\psi_a\psi_b\psi_c\psi_d\rangle &\sim P_{ab}\,P_{cd} + P_{ac}\,P_{bd} + P_{ad}\,P_{bc} + Q_{abcd} \end{aligned}$$

- Closure approximation: $Q_{abcd} = 0$

$$\begin{aligned} B_{abc} = {} & g_{ad}g_{be}g_{cf}\,B_{abc}(\eta = 0) \\ & + 2\int_0^\eta d\eta' e^{\eta'} g_{ad}g_{be}g_{cf} \\ & \times \left[\gamma_{dgh}\,P_{eg}P_{fh} + \gamma_{egh}P_{fg}P_{dh} + \gamma_{fgh}P_{dg}P_{eh}\right] \end{aligned}$$

- Linear propagator g_{ab}:
  $\rightarrow$ describes η-evolution of linear perturbations
  $\rightarrow$ depends via Ω_{ab} on cosmological model

Bispectrum consistency relations

For a flat universe (EdS, ΛCDM):

$$P_{ab}(k) \simeq u_a u_b \, P^L(k) \qquad \text{with } u_a = (1,1)$$

- At linear order:

$$B^L_{abc}{}^{\text{av}}_{q\to 0} \simeq u_b \left(\frac{1}{21} \begin{pmatrix} 47 & 39 \\ 39 & 31 \end{pmatrix}_{ac} - \frac{1}{3} u_a u_c \, k \, \partial_k \right) P^L(k) \, P^L(q)$$

$\rightarrow$ $B^L_{111}{}^{\text{av}}_{q\to 0} \sim \langle \delta\delta\delta \rangle^{\text{av}}_{q\to 0}$:
Coincides with SPT. Reproduces KPRV relation for $P(k) \simeq P^L(k)$

$$\langle \delta\delta\delta \rangle^{\text{av}}_{q\to 0} = \left(\frac{47}{21} - \frac{1}{3} k \, \partial_k \right) P(k) \, P^L(q)$$

[Kehagias, Perrier, Riotto '13], [Valageas '13]

Bispectrum consistency relations

For general cosmological models and all correlations of δ and θ

$$
\begin{aligned}
B_{abc}{}^{\text{av}}_{q\to 0} = \frac{1}{3}\int_0^{\eta} d\eta' e^{\eta'}\, g_{be} \\
\times \Big\{ g_{af}\Big(3\, g_{c1}\, \big[P_{e1}(q)\, P_{f2}(k) + P_{e2}(q)\, P_{f1}(k)\big] \\
+ 2\, g_{c2}\, P_{e2}(q)\, P_{f2}(k)\Big) \\
+ g_{cf}\Big([3\, g_{a1} P_{e1}(q) - g_{a2} P_{e2}(q)]\, P_{f2}(k) \\
- 2\, g_{ad} P_{e2}(q)\; k\,\partial_k P_{fd}(k)\Big\}
\end{aligned}
$$

Bispectrum consistency relations

For a flat universe (EdS, ΛCDM):

- At 1-loop order:

$$B_{111}^{\text{1-loop}} \underset{q\to 0}{\text{av}} \simeq \left[k^2 \left(\alpha + \beta\, k\, \partial_k \right) P^L(k) \times \int dl\, l^2 \left(\frac{P^L(l)}{l^2} \right) + k^4\, \gamma \times \int dl\, l^2 \left(\frac{P^L(l)}{l^2} \right)^2 \right] P^L(q)$$

for $l \gg k \gg q$

→ Coefficients α, β, γ:
Deviation of $\mathcal{O}(10^{-2})$ between SPT and KPRV
⇒ KPRV relation not valid beyond linear order

Curved Background Method

- Non perturbative, but unmeasurable CR: Baldauf et al. 2011

$$B(\boldsymbol{k},-\boldsymbol{q},\boldsymbol{q}-\boldsymbol{k},\eta)^{\rm av} \quad \xrightarrow{q\to 0} \quad P_L(q,\eta)\left[\left(1-\frac{1}{3}k\,\partial_k-\frac{1}{3}\partial_\eta\right)P(k,\eta)+\frac{5}{3}\,\frac{\partial}{\partial\kappa}P_K(k,\eta)\Big|_{K=0}\right],$$

$$\mathrm{VKPR}: \quad \frac{\partial}{\partial\kappa}P_K(k,\eta)\Big|_{K=0} = \frac{4}{7}\,\partial_\eta P_{K=0}(k,\eta)\,.$$

- Fluid equations $\partial_\eta\psi_a = -\Omega_{ab}\,\psi_b + \gamma_{abc}\,\psi_b\psi_c$
- Judicious change of variables:

$$\tilde{\Omega}_{ab} \simeq \Omega_{ab}(\kappa=0) + \frac{3\kappa}{14}\begin{pmatrix} 0 & 0 \\ 1 & -1 \end{pmatrix},$$

The new contribution annihilates the growing mode $\psi_a^{(n)} \propto (1,1)$ for all $n \Rightarrow$ Very good accuracy.