# Black Hole Formation and Classicalization in Trans-Planckian Scattering

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with G. Dvali, C. Gómez, D. Lüst, and S. Stieberger

Reinke Sven Isermann
Ludwig-Maximilians-Universität

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# Einstein Gravity

Einstein gravity ($m = 0$, $s = 2$) is a well-studied theory of gravitation,

- Many interesting features (Geometry, Black Holes, Symmetries, Relation to Yang-Mills,...),
- Supersymmetric extensions,
- Well-tested experimentally (GPS,...)
- ...

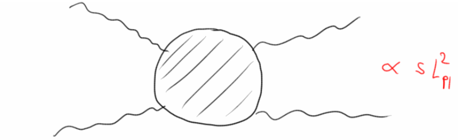
Fortunately for us: many problems and properties still not completely understood.

- UV completion at tree level? Unitarity?
- Quantum understanding of BH?
- (Renormalizability at loop level?)
- ...

# Unitarity at Tree Level in Gravity

Known: Gravity scattering amplitudes grow like $s$ (center of mass energy)

$\Rightarrow$ **violation of (perturbative) unitarity at** $s = M_P^2$.




**Wilsonian UV completion**: regulate by integrating-in weakly-coupled degrees of freedom of shorter and shorter wave-lengths.

**Consequences for gravity**: at energies $s > M_P^2$ UV-completion achieved by new quantum degrees of freedom of wavelength shorter than Planck length.

# UV Completion and Classicalization

**But:** Gravity has a smallest length scale – **the Planck length** (area actually). Cannot go beyond this length since **black holes** will inevitably form, i.e. Wilsonian UV completion does not make sense anymore.

Based on this [Dvali, Gómez] argued that gravity is UV complete by itself through classical black hole formation – called **classicalization**.

Basic idea of UV completion by classicalization is that

$$\textbf{short-scale UV physics} \rightarrow \textbf{long-scale IR physics}$$

by formation of classical object at large energies – **black holes dominate**

In other words: gravity protects itself at high energies by BH formation.

**Without doubt: better quantum understanding of black holes needed.**

# Black Hole $N$ Portrait

Developments towards this in a program of work entitled **Quantum Black Hole corpuscular $N$-portrait**.

[Dvali, Gómez], [Dvali, Gómez, Kehagias], [Dvali, Gómez, Lüst]

Quantum black hole

=

collection of $N$ self-bound gravitons at quantum critical point

(Bose-Einstein condensate)

- interaction strength of gravitons $\alpha = \frac{1}{N}$ at this point
- BH fully characterized by the number $N$
- BH mass $M_{BH} = \sqrt{N} M_P$, BH radius $R_{BH} = \sqrt{N} L_P$, entropy $S = N$
- Black hole physics $\rightarrow$ condensed matter physics

# Black Hole $N$ Portrait

Reproduce **semi-classical behavior** via mean-field approximation

$$N \to \infty \quad \text{and} \quad L_p \to 0 \quad \text{with} \quad \hbar \neq 0$$

Used to pinpoint **quantum origin** of semi-classical properties:

- Bekenstein entropy $\leftrightarrow$ quantum degeneracy of states at critical point
- Hawking radiation $\leftrightarrow$ quantum depletion and leakage of condensate

Can think about classicalization as **large N quantum physics**.

# UV Completion, Classicalization, and the $N$ portrait

Consequently: there are **two interconnected claims**:

- Einstein gravity is UV complete by classicalization (i.e. black hole formation) at tree level
- Black holes are a Bose-Einstein graviton condensate at a quantum critical point

In the language of classicalization and $N$ portrait:

- Black hole formation process should correspond to graviton scattering

$$2 \to N \quad \text{with} \quad p_{in} \sim \sqrt{s} \quad \text{and} \quad p_{out} \sim \sqrt{s}/N \quad \text{with} \quad N \gg 1$$

  via

$$A_{BH} \sim \sum_j |\langle 2|S|N\rangle|^2_P |\langle N|BH\rangle|^2_{NP} \text{ with } |\langle N|BH\rangle|^2_{NP} \sim \exp\{N\}$$

- Moreover, black hole formation should be dominating
- Need to **supplement** perturbative result with non-perturbative input.

# This Talk

Investigate the question of **UV completion** and **black hole formation** in (Einstein) gravity at tree level.

Input from **classicalization** and the $N$-**portrait**

VS

High energy behavior of **scattering amplitudes** in relevant kinematics.

# This Talk

**Plan of the talk:**

1.) Non-perturbative input from the $N$-portrait

2.) Scattering amplitudes in FT and ST at high energies

3.) Interpretation of high energy behavior in light of $N$-portrait

4.) Some further observations / comments

# 1.) Non-perturbative Input from the $N$-portrait

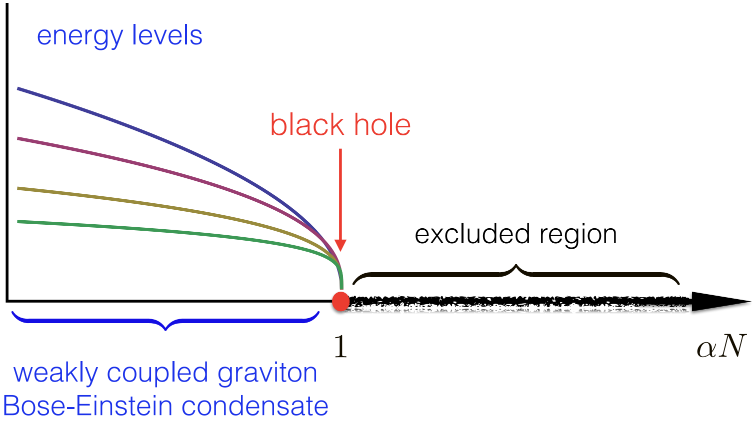
# Black Hole $N$ Portrait: Regimes of $\alpha N$

Different regimes of $\alpha N$ (i.e. the self coupling of the graviton condensate)

- $\alpha N = 1$ black hole formation: **exponential degeneracy of states** ($N$ Bogolyubov modes become gapless) $\sim \exp\{N\}$.
- $\alpha N < 1$ free graviton Bose gas: can be approximated by perturbative methods. **No exponential degeneracy.**
- $\alpha N > 1$ **unphysical region**: Excluded, not a viable $S-matrix$ state (Bogolyubov frequencies complex $\rightarrow$ positive Lyapunov exponents). Region where unitarity would be violated.

# Black Hole $N$ Portrait: Regimes of $\alpha N$

Different regimes of $\alpha N$ (i.e. the self coupling of the graviton condensate)



# 2.) Scattering amplitudes in FT and ST at high energies

# How to actually compute amplitudes in gravity?

Textbook approach: scattering amplitudes $= \sum$ Feynman diagrams.
However: Feynman rules of gravity horribly complicated!

$$
\begin{aligned}
&\frac{\delta^3 S}{\delta\varphi_{\mu\nu}\delta\varphi_{\sigma'\tau'}\delta\varphi_{\rho''\lambda''}} \rightarrow \\
&\quad \mathrm{Sym}\big[-\tfrac{1}{4}P_3(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\lambda})-\tfrac{1}{4}P_6(p^\sigma p^\tau\eta^{\mu\nu}\eta^{\rho\lambda})+\tfrac{1}{4}P_3(p\cdot p'\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\rho\lambda})+\tfrac{1}{2}P_6(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\rho}\eta^{\tau\lambda})+P_3(p^\sigma p^\lambda\eta^{\mu\nu}\eta^{\tau\rho}) \\
&\quad -\tfrac{1}{2}P_3(p^\tau p'^\mu\eta^{\nu\sigma}\eta^{\rho\lambda})+\tfrac{1}{2}P_3(p^\rho p'^\lambda\eta^{\mu\sigma}\eta^{\nu\tau})+\tfrac{1}{2}P_6(p^\rho p^\lambda\eta^{\mu\sigma}\eta^{\nu\tau})+P_6(p^\sigma p'^\lambda\eta^{\tau\mu}\eta^{\nu\rho})+P_3(p^\sigma p'^\mu\eta^{\tau\rho}\eta^{\lambda\nu}) \\
&\qquad\qquad -P_3(p\cdot p'\eta^{\nu\sigma}\eta^{\tau\rho}\eta^{\lambda\mu})\big], \qquad (2.6)
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\delta^4 S}{\delta\varphi_{\mu\nu}\delta\varphi_{\sigma'\tau'}\delta\varphi_{\rho''\lambda''}\delta\varphi_{\iota'''\kappa'''}} \rightarrow \\
&\quad \mathrm{Sym}\big[-\tfrac{1}{8}P_6(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\lambda}\eta^{\iota\kappa})-\tfrac{1}{8}P_{12}(p^\sigma p^\tau\eta^{\mu\nu}\eta^{\rho\lambda}\eta^{\iota\kappa})-\tfrac{1}{4}P_6(p^\sigma p'^\mu\eta^{\nu\tau}\eta^{\rho\lambda}\eta^{\iota\kappa})+\tfrac{1}{8}P_6(p\cdot p'\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\rho\lambda}\eta^{\iota\kappa}) \\
&\quad +\tfrac{1}{4}P_6(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\iota}\eta^{\lambda\kappa})+\tfrac{1}{4}P_{12}(p^\sigma p^\tau\eta^{\mu\nu}\eta^{\rho\iota}\eta^{\lambda\kappa})+\tfrac{1}{2}P_6(p^\sigma p'^\mu\eta^{\nu\tau}\eta^{\rho\iota}\eta^{\lambda\kappa})-\tfrac{1}{4}P_6(p\cdot p'\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\rho\iota}\eta^{\lambda\kappa}) \\
&\quad +\tfrac{1}{4}P_{24}(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\rho}\eta^{\tau\lambda}\eta^{\iota\kappa})+\tfrac{1}{4}P_{24}(p^\sigma p^\tau\eta^{\mu\rho}\eta^{\nu\lambda}\eta^{\iota\kappa})+\tfrac{1}{4}P_{12}(p^\rho p'^\lambda\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\iota\kappa})+\tfrac{1}{2}P_{24}(p^\sigma p'^\rho\eta^{\tau\mu}\eta^{\nu\lambda}\eta^{\iota\kappa}) \\
&\quad -\tfrac{1}{2}P_{12}(p\cdot p'\eta^{\nu\sigma}\eta^{\tau\rho}\eta^{\lambda\mu}\eta^{\iota\kappa})-\tfrac{1}{2}P_{12}(p^\sigma p'^\mu\eta^{\tau\rho}\eta^{\lambda\nu}\eta^{\iota\kappa})+\tfrac{1}{2}P_{12}(p^\sigma p^\rho\eta^{\tau\lambda}\eta^{\mu\nu}\eta^{\iota\kappa})-\tfrac{1}{2}P_{24}(p\cdot p'\eta^{\mu\nu}\eta^{\tau\rho}\eta^{\lambda\iota}\eta^{\kappa\sigma}) \\
&\quad -P_{12}(p^\sigma p^\tau\eta^{\nu\rho}\eta^{\lambda\iota}\eta^{\kappa\mu})-P_{12}(p^\rho p'^\lambda\eta^{\nu\iota}\eta^{\kappa\sigma}\eta^{\tau\mu})-P_{24}(p_\sigma p'^\rho\eta^{\tau\iota}\eta^{\kappa\mu}\eta^{\nu\lambda})-P_{12}(p^\rho p'^\iota\eta^{\lambda\sigma}\eta^{\tau\mu}\eta^{\nu\kappa}) \\
&\quad +P_6(p\cdot p'\eta^{\nu\rho}\eta^{\lambda\sigma}\eta^{\tau\iota}\eta^{\kappa\mu})-P_{12}(p^\sigma p^\rho\eta^{\mu\nu}\eta^{\tau\iota}\eta^{\kappa\lambda})-\tfrac{1}{2}P_{12}(p\cdot p'\eta^{\mu\rho}\eta^{\nu\lambda}\eta^{\sigma\iota}\eta^{\tau\kappa})-P_{12}(p^\sigma p^\rho\eta^{\tau\lambda}\eta^{\mu\iota}\eta^{\nu\kappa}) \\
&\qquad\qquad -P_6(p^\rho p'^\iota\eta^{\lambda\kappa}\eta^{\mu\sigma}\eta^{\nu\tau})-P_{24}(p^\sigma p'^\rho\eta^{\tau\mu}\eta^{\nu\iota}\eta^{\kappa\lambda})-P_{12}(p^\sigma p'^\mu\eta^{\tau\rho}\eta^{\lambda\iota}\eta^{\kappa\nu})+2P_6(p\cdot p'\eta^{\nu\sigma}\eta^{\tau\rho}\eta^{\lambda\iota}\eta^{\kappa\mu})\big]. \qquad (2.7)
\end{aligned}
$$

[DeWitt]

# How to compute amplitudes in gravity?

Example: 4 points tree level. Feynman diagrams give $\mathcal{O}(100)$ terms.
**Result extremely simple:**

$$M(1^-, 2^-, 3^+, 4^+) = \frac{\langle 12\rangle^7 [12]}{\langle 13\rangle\langle 14\rangle\langle 23\rangle\langle 24\rangle\langle 34\rangle^2}$$

with $[ij]$ and $\langle ij\rangle$ roughly $\sim \sqrt{|s_{ij}|}$ (spinor helicity formalism).

What's the meaning of this?

- Huge cancellations in sum over terms; Feynman diagrams **not** the correct way to compute (offshell, unphysical information!)
- Missed a symmetry?
- Alternative methods?

# KLT relations (1986) [Kawai, Lewellen, Tye]

- $N$ graviton amplitude $\sim$ sum of squares of $N$ gluon amplitudes
- Can be derived most easily in string theory
  (closed string $\sim$ open string $\times$ open string)

$$M_N = \left(-\frac{\kappa}{2}\right)^{N-2} \sum_{\sigma,\gamma\in S_{N-3}} A_N(1,\sigma(2,...,N-2),N-1,N)$$
$$S[\gamma(2,...,N-2),\sigma(2,...,N-2)]_{N-1} A_N(1,N-1,\sigma(2,...,N-2),N)$$

[Bern, Dixon, Perelstein, Rozowsky]
[Bjerrum-Bohr, Damgaard, Sondergaard, Vanhove]

- $S[...,...]$ called *momentum kernel*. Roughly $S \sim s_{ij}^{N-3}$
- $A_N(...)$ *color-ordered* Yang-Mills amplitude
- Example:

$$M_4 = s_{12} A_4(1,2,3,4) A_4(2,1,3,4)$$

# Scattering Equations (2013) [Cachazo, He, Yuan]

- Tree-level S-matrix of massless particles with spin 0,1,2 (and also mixed amplitudes) in arbitrary spacetime dimension given by integral over punctures on a sphere.

$$M_{N,s} = \int \frac{d^N\sigma}{\text{vol } SL(2,\mathbb{C})} \prod_a{}' \delta\Big(\sum_{b\neq a} \frac{s_{ab}}{\sigma_a - \sigma_b}\Big) \left( \frac{Tr(T^{a_1} \ldots T^{a_N})}{(\sigma_1 - \sigma_2)(\sigma_2 - \sigma_3)\ldots} + \ldots \right)^{2-s} (Pf'\Psi)^s$$

- Kinematic part independent of theories, given by a system of equations called **scattering equations**

$$\sum_{b=1, b\neq a}^{n} \frac{s_{ab}}{\sigma_a - \sigma_b} = 0, \quad a = 1, \ldots, n$$

- (N-3)! solutions to these equations determine position of $n$ points on sphere, localizes integral.
- **Caveat:** *extremely hard to solve in general for arbitrary kinematics.*

# Classicalization Regime

- Energy regime in $2 \rightarrow N$ scattering according to classicalization corresponds to

$$p_{in} \sim \sqrt{s} \qquad \text{and} \qquad p_{out} \sim \frac{\sqrt{s}}{N}$$

$$\Rightarrow \quad s_{ij} = (p_i + p_j)^2 \sim \begin{cases} s, & \{i,j\} \in \{1,N\} \\ -\frac{s}{N}, & i \in \{1,N\},\ j \notin \{1,N\} \\ \frac{s}{N^2}, & \{i,j\} \notin \{1,N\} \end{cases}$$

- Defined particles 1 and $N$ incoming, rest outgoing.

# Classicalization Regime and Scattering Equations

Rewrite this regime as (in units of $\frac{s}{N^2}$) introducing two parameter $\mathfrak{a}, \mathfrak{b}$, with $-1 < \mathfrak{a}, \mathfrak{b} < 0$, s.t.

$$s_{1,N} = \frac{1}{2}(N - \mathfrak{a} - \mathfrak{b}) \quad s_{ij} = 1, \quad i,j = \{2, ...N-2\}$$
$$s_{N-1,N} = -\frac{1}{2}(N-3)(2-\mathfrak{b}) \quad s_{1,N-1} = -\frac{1}{2}(N-3)(2-\mathfrak{a})$$
$$s_{1,i} = -\frac{1}{2}(N-2-\mathfrak{b}) \quad s_{i,N} = -\frac{1}{2}(N-2-\mathfrak{a})$$
$$s_{N-1,i} = \frac{1}{4}(4-\mathfrak{a}-\mathfrak{b})$$

- Similar setup studied by [Kalousios]:
- Solutions degenerate: only $(N-3)$ instead of $(N-3)!$ indep. sold.
- Solutions to scattering eqs = zeros of Jacobi polynomials $P_{N-3}^{(\mathfrak{a},\mathfrak{b})}$
- $P_n^{(\alpha,\beta)}(x) = \frac{(-1)^n}{2^n n!}(1-x)^{-\alpha}(1+x)^{-\beta}\frac{d^n}{dx^n}\left[(1-x)^{\alpha+n}(1+x)^{\beta+n}\right].$

# Classicalization Regime

Use Kalousios' insights to obtain $N$-point gravity amplitude in classicalization regime:

$$M_N = -\kappa^{N-2}\, 2^{8-N}\, \frac{s}{N^2}\, [(N-3)!!]^2\, \frac{\Gamma\left(\frac{\mathfrak{a}}{2}\right)\ \Gamma\left(\frac{3}{2}+\frac{\mathfrak{b}-N}{2}\right)\ \Gamma\left(\frac{1-N+\mathfrak{a}+\mathfrak{b}}{2}\right)}{\Gamma\left(1+\frac{\mathfrak{a}-N}{2}\right)\ \Gamma\left(\frac{\mathfrak{b}-1}{2}\right)\ \Gamma\left(\frac{\mathfrak{a}+\mathfrak{b}-3}{2}\right)}$$
$$\times \frac{\Gamma\left(\frac{3}{2}+\frac{\mathfrak{a}-N}{2}\right)\ \Gamma\left(\frac{\mathfrak{b}}{2}\right)\ \Gamma\left(\frac{\mathfrak{a}+\mathfrak{b}-2}{2}\right)}{\Gamma\left(1+\frac{\mathfrak{b}-N}{2}\right)\Gamma\left(\frac{\mathfrak{a}-1}{2}\right)\ \Gamma\left(\frac{\mathfrak{a}+\mathfrak{b}-N}{2}\right)}\ H_N(\mathfrak{a},\mathfrak{b})^2$$

with $H_N(\mathfrak{a},\mathfrak{b})$ encoding polarisation (but constant in $N$).

For $N \gg 1$ Taylor expand and find

$$M_N \sim \kappa^N\, \frac{s}{N^2}\, N!$$

# Graviton Scattering Amplitudes in Classicalization Regime

To obtain the physical probability i.e. the S-matrix element, have to consider

$$d|\langle 2|S|N\rangle|^2 \sim \frac{1}{(N)!} \prod_{i=2}^{N-1} dp_i^4 |M_N|^2 \delta^4(P_{total})$$

(Full cross section by integrating over momenta and summing over helicities)

Plugging in classicalization regime gives (taking $N \gg 1$, $\kappa = L_P$, and Stirling's formula)

$$\boxed{|\langle 2|S|N\rangle|^2 \sim \left(\frac{L_P^2 s}{N^2}\right)^N N! \sim \exp(-N)\ \lambda^N}$$

Define $\lambda = \frac{L_P^2 s}{N}$ for later convenience (collective coupling).

# String Amplitudes

Known: High Energy behavior of open and / or closed string amplitudes given by exponential fall-off. [Veneziano], [Gross, Mende], [Gross, Manes]

Thus no problem with unitarity at transplanckian energies.

- Example: 4-point closed string amplitude for $\alpha' \to \infty$

$$\mathcal{M}_4 \sim \kappa^2 |A_4|^2 \times 4\pi\alpha' \, \frac{st}{u} \, \exp\left\{ \frac{\alpha'}{2} (s \ln |s| + t \ln t + u \ln u) \right\}$$

- Note: State-of-the-art until our paper came out!
- Computation via Laplace's *saddle point method* on world-sheet integrals:

$$\int g(x) \exp\{\alpha' f(x)\} dx \sim \sqrt{\frac{2\pi}{\alpha' |f''(x_0)|}} g(x_0) \exp\{\alpha' f(x_0)\} + \mathcal{O}(\alpha'^{-1})$$

with $x_0$ unique global maximum in interval of integration.

# High Energy Behavior of $N$-point String Amplitudes

- Shall see: High energy string behavior closely related to scattering equations and their solutions
- Generic (open string) Koba-Nielsen factor given by

$$Z \sim \int \prod_i dz_i \prod_{i<j}^{N} |z_{ij}|^{\alpha' s_{ij}}$$

- Koba-Nielsen factor can be written as

$$\prod_{i<j}^{N} z_{ij}^{\alpha' s_{ij}} = \exp\left\{ \frac{\alpha'}{2} \sum_{i \neq j} s_{ij} \ln|z_i - z_j| \right\}$$

- Then: condition for saddle point = scattering equations

$$\sum_{j \neq i} \frac{s_{ij}}{z_i - z_j} = 0, \quad i = 1, ..., N \quad \text{has } (N-3)! \text{ solutions in general}$$

# High Energy Behavior of $N$-point Closed String Amplitudes

Based on scattering equations, leading term of N-point closed string for $\alpha' \to \infty$ can be written as [CHY]

$$\mathcal{M}_N = \kappa^{N-2} \, (4\pi\alpha')^{N-3} \sum_{a=1}^{(N-3)!} \frac{\left( \prod_{i<j}^{N} |z_{ij}^{(a)}|^{\frac{\alpha'}{2} s_{ij}} \right)}{\det{}' \, \Phi(z^{(a)})^{1/2} \det{}' \, \Phi(\bar{z}^{(a)})^{1/2}} \, E_N(\{k, \xi, z^{(a)}\})^2 + \mathcal{O}(\alpha'^{-1})$$

- Sum runs over solutions to scattering equations
- $E_N$ encodes momenta and polarizations
- $\det' \Phi$ comes from localizing the integrations $\delta(f(x)) = \sum_i \frac{\delta(x - x_i)}{|f'(x_i)|}$
- Note how high energy limit of string theory amplitude looks very similar to field theory amplitude. Still not understood.

- Work out Koba-Nielsen factor above in classicalization regime now...

# Properties of Zeros of Jacobi Polynomials

Have seen close relationship to zeros of Jacobi polynomials $P_{N-3}^{(\alpha\beta)}(x)$.
Study their properties [Szegö]:

(**1.**) **Discriminant** of Jacobi polynomials given by

$$\Delta_{N-3} := l^{2N-8} \prod_{1\leq a<b\leq N-3} (x_a - x_b)^2$$

$$= \frac{1}{2^{(N-3)(N-4)}} \prod_{\nu=1}^{N-3} \frac{(\alpha+\nu)^{\nu-1}\,(\beta+\nu)^{\nu-1}\,(\alpha+\beta+N-3+\nu)^{N-3-\nu}}{\nu^{-(\nu-2N+8)}}$$

with $l$ is coefficient of highest term $x^{N-3}$ of Jacobi polynomial $P_{N-3}^{(\alpha\beta)}(x)$.

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(**2.**) $$\prod_{a=1}^{N-3}(1-x_a) = (N-3)!\,\frac{P_{N-3}^{(\alpha\beta)}(1)}{P_{N-3}^{(\alpha\beta)^{(N-3)}}(x)} = 2^{N-3} \prod_{\nu=1}^{N-3} \frac{(\alpha+\nu)}{(\alpha+\beta+N-3+\nu)}$$

(**3.**) $$\prod_{a=1}^{N-3}(1+x_a) = (-1)^{N+1}(N-3)!\,\frac{P_{N-3}^{(\alpha\beta)}(-1)}{P_{N-3}^{(\alpha\beta)^{(N-3)}}(x)} = 2^{N-3} \prod_{\nu=1}^{N-3} \frac{(\beta+\nu)}{(\alpha+\beta+N-3+\nu)}$$

# Koba-Nielsen Factors on Solutions of Scattering Equation

Solutions of the scattering equation in the classicalization regime given by permutation $\pi_I \in S_{N-3}, I = 1, ..., (N-3)!$ acting on the $N-3$ zeros $x_a$ via $\{z_i^I = x_{\pi_{I(i-1)}} \mid i = 2, ..., N-2\}$. Gauge fix $z_1^{(I)} = -1,\ z_{N-1}^{(I)} = \infty,\ z_N^{(I)} = 1$.

$$\prod_{i<j} |z_{ij}^{(I)}|^{\alpha' s_{ij}} = 2^{\alpha' s_{1N}} \prod_{a=2}^{N-2} |z_1^{(I)} - z_a^{(I)}|^{\alpha' s_{1a}}\ |z_N^{(I)} - z_a^{(I)}|^{\alpha' s_{aN}} \prod_{2\leq a<b\leq N-2} |z_a^{(I)} - z_b^{(I)}|^{\alpha' s_{ab}}$$

$$= \prod_{\nu=1}^{N-3} \left( \frac{\nu^\nu\ (\alpha+\nu)^{\alpha+\nu}\ (\beta+\nu)^{\beta+\nu}}{(\alpha+\beta+N-3+\nu)^{\alpha+\beta+N-3+\nu}} \right)^{\alpha'/2}$$

- Note that independent on which permutation under consideration. Each solution yields same Koba-Nielsen factor.

# $N$-point Closed String Amplitude in Classicalization Regime

- $N$-point closed tree amplitude becomes (permutation invariance)

$$\mathcal{M}_N = \kappa^{N-2}\ (4\pi\alpha')^{N-3}\ \left(\prod_{i<j}^{N} |z_{ij}^{(a)}|^{\frac{\alpha'}{2}s_{ij}}\right) \sum_{a=1}^{(N-3)!} \frac{E_N(\{k,\xi,z^{(a)}\})^2}{\det{}'\,\Phi(z^{(a)})^{1/2}\det{}'\,\Phi(\overline{z}^{(a)})^{1/2}}$$

  - Solutions here real: $\det{}'\,\Phi(z^{(a)})^{1/2}\det{}'\,\Phi(\overline{z}^{(a)})^{1/2} = \det{}'\,\Phi(z^{(a)})$
  - $\sum_{a=1}^{(N-3)!} \dots = M_N^{FT}$. Computed some slides ago.

**Final result:**

$$\mathcal{M}_N = (4\pi\alpha')^{N-3} \quad \prod_{\nu=1}^{N-3} \left(\frac{\nu^{\nu}\ (\alpha+\nu)^{\alpha+\nu}\ (\beta+\nu)^{\beta+\nu}}{(\alpha+\beta+N-3+\nu)^{\alpha+\beta+N-3+\nu}}\right)^{\alpha'/4} \quad M_N^{FT} + \mathcal{O}(\alpha'^{-1})$$

with $\mathfrak{a} = \alpha + N - 1$ and $\mathfrak{b} = \beta + N - 1$.

# Comments and Further Results

$$\mathcal{M}_N = (4\pi\alpha')^{N-3} \quad \prod_{\nu=1}^{N-3} \left( \frac{\nu^\nu \ (\alpha+\nu)^{\alpha+\nu} \ (\beta+\nu)^{\beta+\nu}}{(\alpha+\beta+N-3+\nu)^{\alpha+\beta+N-3+\nu}} \right)^{\alpha'/4} \ M_N^{FT} + \mathcal{O}(\alpha'^{-1})$$

- Same analysis also holds in pure $\alpha' \to \infty$ regime (i.e. $|s_{ij}| \sim s$).

$$\mathcal{M}_N \sim \kappa^{N-2} \alpha'^{N-3} \ s \ \exp\left\{ -\frac{\alpha'}{2}(N-3) s \ln(\alpha' s) \right\}$$

- Can also consider a regime where $s$ and $N$ are large but $\frac{s}{N^2}$ is below string scale

$$\mathcal{M}_N \to M_N^{FT}$$

Conjectured by [Cheung, O'Connell, Wecht].

- Similar analyses can be done for open string tree amplitudes; structure of results very similar (s. paper)

# 3.) Interpretation of High Energy Behavior

# Interpretation of High Energy Behavior in light of $N$-portrait & Classicalization

Field theory result:

$$|\langle 2|S|N\rangle|^2 \sim \left(\frac{\lambda}{N}\right)^N N! \sim \exp(-N)\ \lambda^N \quad , \qquad \lambda \equiv \alpha N = \frac{L_P^2 s}{N}$$

- Remember that in $N$-portrait: $\lambda = \alpha N$ and $\alpha N > 1$ not allowed (unitarity violation).
- At $\lambda = 1$, amplitude $\sim \exp\{-N\}$ but has to be supplemented by bh degeneracy of states $\Rightarrow$ compensation

$$A_{BH} \sim |\langle 2|S|N\rangle|^2 |\langle N|BH\rangle|^2_{NP} \sim \left(\frac{1}{N}\right)^N N! \times \exp N \sim 1$$

- Close to $\lambda \lesssim 1$, degeneracy of states still countable, but another suppression $\sim \lambda^N$ factor which is not compensated for.
- $\Rightarrow$ dominance of BH final states over other possible multi-particle final states.

# Interpretation of High Energy Behavior in light of $N$-portrait & Classicalization

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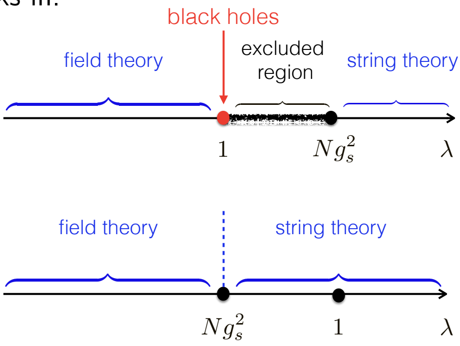
- Behavior of large $\sqrt{s}$ smoothened out if $N$ increases appropriately $\Rightarrow$ core idea of classicalization.
- Smoothing out starts at $N = sL_P^2$. "Unitarity threshold for given $s$".
- In $N$-Portrait this is exactly entropy of a BH of mass $\sqrt{s}$
- Everything above unitarity threshold excluded by corpuscular picture (by black hole formation)

# Interpretation of High Energy Behavior in light of $N$-portrait & Classicalization

Planck and string length related. Identify two regimes:

- $\lambda = g_s^2 N > 1$: string effects relevant where outgoing gravitons strongly coupled. Does it tame unitarity violation in FT?
- $\lambda = g_s^2 N < 1$: string effects become relevant *before* black hole formation kicks in.



4.) Some further observations (and speculations...)

# On the point $g_s^2 N = 1$

Threshold of string effects matches field theoretical critical point of black hole formation.

$$g_s = \frac{1}{\sqrt{N}}$$

- point where string coupling of constituent quanta becomes equally important as gravitational coupling
- corresponds to string-black hole correspondence,i.e.
  black hole state $\sim$ state of strings and D-branes with same charges

[Horowitz, Polchinski], [Dvali, Gómez], [Dvali, Lüst]

# On $GR = YM^2$

Gravity amplitudes can be expressed as sum over Yang Mills amplitudes squared. Known for a long time, basis for many developments like recent study of UV properties of $\mathcal{N} = 8$ by [Bern et al] up to 5 loops.

- But: never used at any point information about color of Yang-Mills $N_c$
- Connection closed string open string coupling:

$$g_s = g_{open}^2$$

- At point of string-bh correspondence:

$$g_s = \frac{1}{\sqrt{N}}$$

- ['t Hooft]:

$$g_{open}^2 = \frac{1}{N_c}$$

Thus naively: $N = N_c^2$ Interpretation?

# Summary

- Studied high energy behavior of graviton amplitudes at tree level.
- Established connection between transplanckian scattering amplitudes and unitarization by BH formation (classicalization).
- Used classicalization and the BH corpuscular $N$ portrait as a guide.

**Findings in Field theory:**

- Closed expressions for tree-level $N$-point graviton and gluon amplitudes in classicalization regime
- Identify microscopic reason of BH dominance over other final states.
- Find that high-energy behavior of graviton FT amplitudes becomes smoothened out when number $N$ of produced gravitons is increased.
- Unitarity threshold at $N = sL_P^2$ for given $s$. Corresponds in $N$ portrait to BH of mass $\sqrt{s}$.
- Strong coupling regime excluded by corpuscular arguments.

# Summary

- Studied high energy behavior of graviton amplitudes at tree level.
- Established connection between transplanckian scattering amplitudes in FT and ST and unitarization by BH formation (classicalization).
- Used classicalization and the BH corpuscular $N$ portrait as a guide.

**Findings in String theory:**

- Closed expressions for tree-level $N$-point open and closed string scattering at high energies.
- Beautiful connection to recent developments in FT (*scattering equations*)
- Identify two regimes (not talked about today in detail)
  - $\frac{\sqrt{s}}{N} < M_s$: String amplitudes agree with FT amplitudes at $N$-points
  - $\frac{\sqrt{s}}{N} > M_s$: String effects become important

# Summary

- Could identify interplay between $N$ portrait, black hole formation, and scattering amplitudes in a field theory regime and string regime.

- Amplitudes reveal key features of the $N$ portrait; perturbative amplitudes already seem to know about non-perturbative physics.

# Outlook

- Precise the role of color in "$GR = YM^2$"?
- Implications along the lines of AdS/CFT?
- Beyond tree level in light of classicalization and $N$-portrait? First steps in [Kuhnel, Sandborg].
- Next: High energy behavior of amplitudes including gluons? Take as inspiration [Dvali, Gómez, Lüst] and [Stieberger] (ST) or [Cachazo, He, Yuan] in (FT) – *work in progress.*

**Stay tuned...**

**Your Questions Here?**

# Extra slides

# Scattering Equations [Cachazo, He, Yuan]

Skipping most details, the final formula for the tree-level S-matrix of a massless spin $s$ particle is given by ($\sigma_{kl} = \sigma_k - \sigma_l$)

$$M_{n,s} = \sum_{\{\sigma\}\in sol} \left( \frac{\mathrm{Tr}(T^{a_1} \dots T^{a_n})}{\sigma_{12} \dots \sigma_{n1}} + \text{perms} \right)^{2-s} \frac{(\mathrm{Pf}'\Psi(\{k,\epsilon,\sigma\}))^s}{\det{}'\Phi(\sigma)}$$

with $\Psi$ a $2n \times 2n$ skew-symmetric matrix given by $\Psi = \begin{pmatrix} A & -C^T \\ C & B \end{pmatrix}$ with

$$A_{ab} = \begin{cases} \frac{s_{ab}}{\sigma_{ab}}, & a \neq b \\ 0\,, & a = b \end{cases} \quad B_{ab} = \begin{cases} \frac{2\epsilon_a \cdot \epsilon_b}{\sigma_{ab}}, & a \neq b \\ 0\,, & a = b \end{cases} \quad C_{ab} = \begin{cases} \frac{2\epsilon_a \cdot k_b}{\sigma_{ab}}, & a \neq b \\ -\sum_{c\neq a} \frac{2\epsilon_a \cdot k_c}{\sigma_{acs}}, & a = b \end{cases}$$

and $\mathrm{Pf}'\Psi = \frac{(-1)^{i+j}}{\sigma_{ij}} \mathrm{Pf}(\Psi^{ij}_{ij})$, $1 \leq i < j \leq n$ and $\mathrm{Pf}\,\Psi^{ij}_{ij} = \sqrt{\det\Psi^{ij}_{ij}}$.

$$\det\Phi' \equiv \frac{\det(\Phi)^{rst}_{ijk}}{\sigma_{ij}\sigma_{jk}\sigma_{ki}\sigma_{rs}\sigma_{st}\sigma_{tr}} \text{ with } \Phi_{ab} = \begin{cases} \frac{s_{ab}}{\sigma^2_{ab}}, a \neq b \\ -\sum_{c\neq a} \frac{s_{ac}}{\sigma^2_{ac}}, a = b \end{cases}$$

Closed-string amplitude can be written via (symmetric) KLT relations

$$M_N(\alpha') = \int D^{N-3}_{\alpha'} z_i D^{N-3}_{\alpha'} \bar{z}_i \sum_{\tau,\tilde{\tau},\rho,\tilde{\rho}} \frac{S[\rho|\tau]S[\tilde{\rho}|\tilde{\tau}]}{z_{1,\rho(2)} \cdots z_{N-1,N} z_{N,1} \bar{z}_{1,\tilde{\rho}(2)} \cdots \bar{z}_{N,1}} A_{YM}(\tau) A_{YM}(\tilde{\tau})$$

- $D^{N-3}_{\alpha'} z_i = \frac{d^N z_i}{volSL(2,\mathbb{C})} \prod_{i<j} |z_{ij}|^{\alpha' s_{ij}}$
- $S[...|...]$ momentum kernel and $\tau, \tilde{\tau}, \rho, \tilde{\rho} \in S_{N-3}$

Using and properties of scattering equation amplitudes on $A_{YM}$

$$M_N = \sum_{a=1}^{(N-3)!} \frac{\left( \prod_{i<j}^{N} |z_{ij}^{(a)}|^{\frac{\alpha'}{2} s_{ij}} \right)}{\det{}' \Phi(z^{(a)})^{1/2} \det{}' \Phi(\bar{z}^{(a)})^{1/2}} E_N(\{k, \xi, z^{(a)}\})^2 + \mathcal{O}(\alpha'^{-1}) \ .$$

with $E_N^2 = det' \Psi$.