Sourcing the central charge: Exact results for open strings in AdS/CFT

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Outline

- The one loop spin chain and the central charge.
- Ending on D-branes: giant gravitons.
- Boundary conditions: one and two loops.
- All loop orders, beta deformations and geometry.

One loop spin chain

States in N=4 SYM on cylinder can be described in terms of gauge invariant local operator insertions.

String states are traces

$$\mathcal{O}(0) \simeq \mathrm{Tr}(W_1 W_2 \ldots)$$

$$W \in (\partial^{[n]}\phi_{1,..6}, \partial^{[n]}\psi^I, \partial^{[n]}F_{\mu\nu})$$

Derivatives should be covariantized.

SU(2) sector

Choose an $N=1$ R-charge, and decompose the multiplet into three chiral super fields X,Y,Z and vector superfield.

The SU(2) sector is built only from Y,Z and no derivatives.

$$\mathcal{O}(0) \simeq \mathrm{Tr}(ZYZZZYYZYZ....)$$

The one loop spin chain is proportional to

$$H_{1-loop} \propto g_{YM}^2 N \mathrm{Tr}[Y, Z][\partial_Z, \partial_Y]$$

Minahan & Zarembo;
Besiert, Christiansen and Studacher.

Infinite spin chain limit

$$\mathcal{O}(0) \simeq zzzzzzzzzY zzzzzzzzzzzzzzz$$

The Z make a ground state, and
the Y hop to left or right as impurity.

There is a (quasi) momentum on spin chain associated to eigenstates of the discrete translation.

When considering bigger sectors with SUSY,

one can associate a central charge to the spin chain (Beisert).

Which shows effectively as follows

$$Y \to [Y, Z]$$

Evaluating one gets

$$\{Q, Q\} \propto \exp(ip)$$

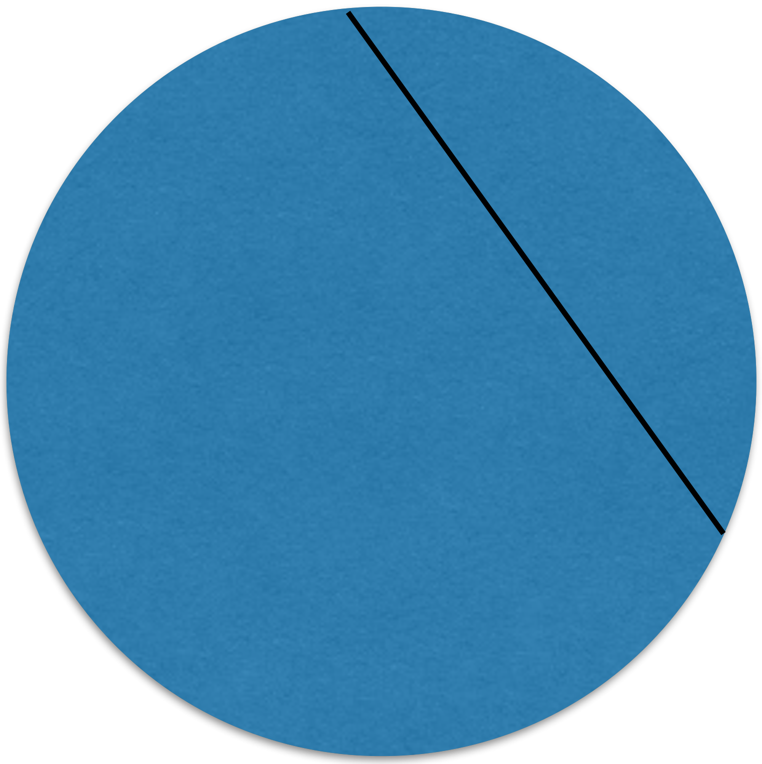
Produces a non-trivial dispersion relation for impurity

$$\Delta - J \simeq \sqrt{1 + h(\lambda)\sin^2 p/2}$$

Good to “all orders”

$$h(\lambda) = \lambda$$

This is the strong ’t Hooft coupling constant.



This is the dispersion relation
of a giant magnon: a straight chord
between two end-points on a disk,
moving with angular velocity one

(Hofmann -Maldacena)

$$\begin{array}{ccc} S^3 & \rightarrow & S^5 \\ & & \downarrow \\ & & D_2 \end{array}$$

Can form BPS bound states of k such impurities

$$\Delta - J = \sqrt{k^2 + h(\lambda)\sin^2 p/2}$$

N. Dorey.

- Central charge is confined in closed strings.
- How does it relate to central charge of N=4 SYM on Coulomb branch?
- In flat 10 D space, central charge can be sourced by placing D-branes in various positions.
- How does one build correct boundary conditions for spin chain from first principles?

Ending on D-branes

Want a D-brane configuration that preserves same symmetries of ground state of spin chain: built out of Z.

There are such D-branes: giant gravitons & dual giant gravitons.

GIANT GRAVITONS

Gravitons: half BPS states of AdS

Point particles moving on a diameter of sphere and sitting at origin of AdS

Preserve SO(4)x SO(4) symmetry

Solution

$$(x^1)^2 + (x^2)^2 + r_{S^3}^2 = 1$$

solving equations of motion gives

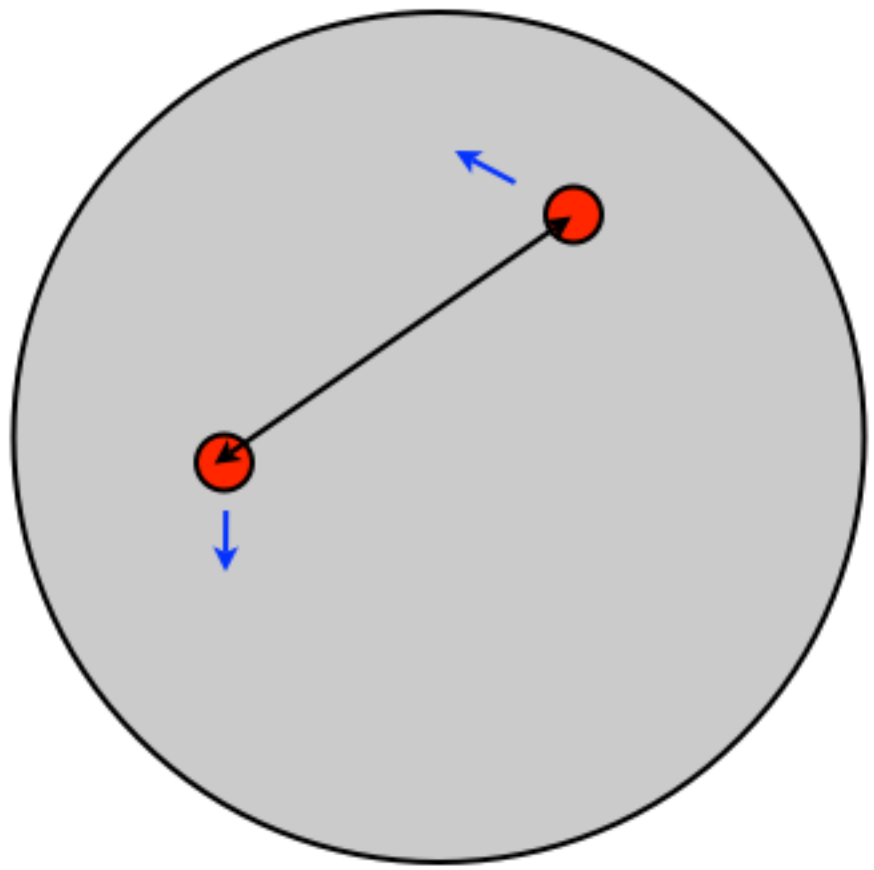
$$x^1 + ix^2 = z = \exp(it)$$

Picture as a point on disk moving with angular velocity one

The one at z=0 has maximum angular momentum

McGreevy, Susskind, Toumbas, hep-th/000307

They are D-branes



Can attach strings

Gauge symmetry
on worldvolume

Gauss' law
Strings in =
Strings out

Mass of strings should be roughly a distance:
depends on geometric position of branes

In gravity, D-branes are localized, but if they have a fixed R-charge in the quantum theory, they are delocalized in the angle variable of z

This is, they correspond to a oscillating wave function on the angle of z (zero mode)

To find masses of strings the branes must also be localized on angles, so they require uncertainty in angular momentum.

Giant graviton states:

$$\det_\ell Z = \frac{1}{N!}\binom{N}{\ell}\epsilon_{i_1,\ldots,i_\ell,i_{\ell+1}\ldots,i_N}\epsilon^{j_1,\ldots,j_\ell,i_{\ell+1}\ldots,i_N}Z^{i_1}_{j_1}\ldots Z^{i_\ell}_{j_\ell}$$

Subdeterminant operators

Balasubramanian, Berkooz, Naqvi, Strassler, hep-th/0107119

Complete basis of all half BPS operators in terms of Young Tableaux,

Corley, Jevicki, Ramgoolam, hep-th/0111222

Interpretation

A giant graviton with fixed R-charge is a quantum state that is delocalized in dual variable to R-charge

To build localized states in dual variable we need to introduce a collective coordinate that localizes on the zero mode: need to introduce uncertainty in R-charge

Introduce collective coordinate for giant gravitons

Consider

$$\det(Z-\lambda) = \sum_{\ell=0}^{N} (-\lambda)^{N-\ell} \det_\ell(Z)$$

This is a linear combination of states with different R-charge, depends on a complex parameter, candidate for localized giant gravitons in angle direction

Attaching strings

The relevant operators for maximal giant are

$$\epsilon\epsilon(Z, \ldots Z, W^1, \ldots W^k)$$

Balasubramanian, Huang, Levi and Naqvi, hep-th/0204196

These can be obtained from expanding

$$\det(Z + \sum \xi_i W^i)$$

And taking derivatives with respect to parameters

One loop anomalous dimensions = masses of strings

Want to compute effective Hamiltonian of strings stretched between two giants.

$$\det(Z-\lambda_1)\det(Z-\lambda_2)\mathrm{Tr}((Z-\lambda_1)^{-1}Y(Z-\lambda_2)^{-1}X)$$

Exact full combinatorics of 2 giants on same group is messy: easier to illustrate on orbifolds.

$$H_{1-loop} \propto g_{YM}^2 N \mathrm{Tr}[Y, Z][\partial_Z, \partial_Y]$$

Need following partial results

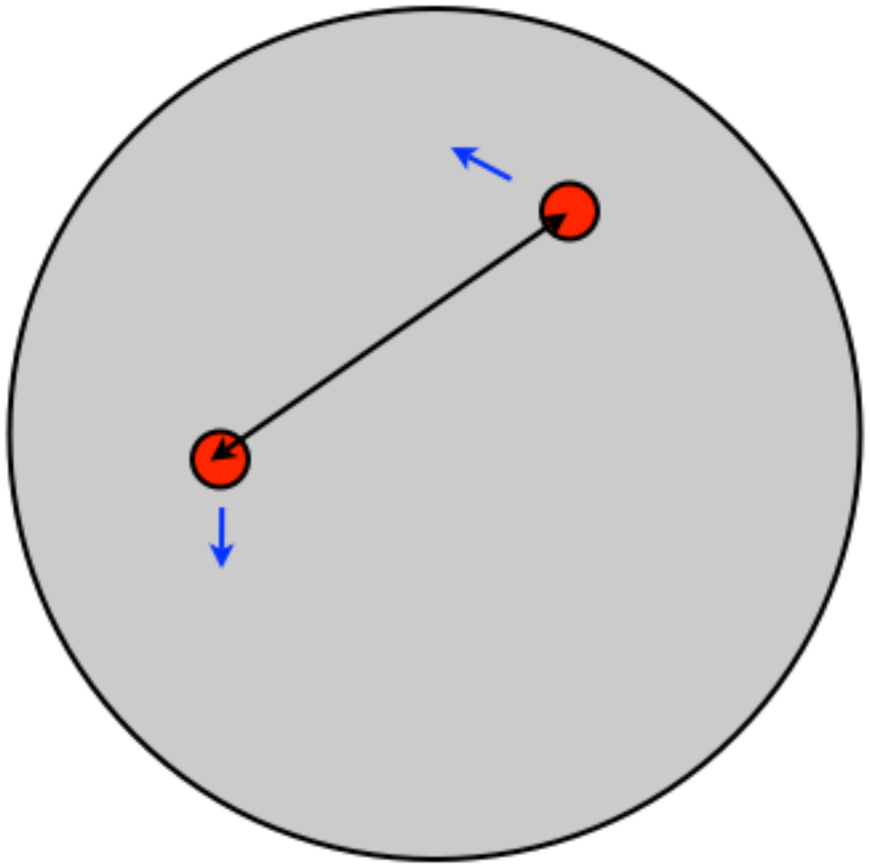
$$\partial_Z \det(Z-\lambda) = \det(Z-\lambda)\frac{1}{Z-\lambda}$$

$$\partial_Z \mathrm{tr}\left((Z-\lambda)^{-1}W\right) = -(Z-\lambda)^{-1}W(Z-\lambda^{-1})$$

Collect planar contributions.

D.B. ArXiv:1301.3519

What we get in pictures



$$m_{od}^2 \simeq g_{YM}^2 |\lambda - \tilde{\lambda}|^2$$

$$E \simeq m_{od}^2 \simeq g_{YM}^2 |\lambda - \tilde{\lambda}|^2$$
$$\simeq g_{YM}^2 N |\xi - \tilde{\xi}|^2$$

Result is local in collective coordinates (terms that could change collective parameters are exponentially suppressed)

Mass proportional to distance is interpreted as Higgs mechanism for emergent gauge theory.

Spin chains

$$Y \to Y^n$$

Need to be careful about planar versus non-planar diagrams.

$$\lambda \simeq N^{1/2}$$

Simplest open chains

$$\det(Z-\lambda)\mathrm{Tr}(\frac{1}{Z-\lambda}YZ^{n_1}Y\ldots Z^{n_k}Y)$$

Just replace the W by n copies of Y: Z can jump in and out at edges. So we need to keep arbitrary Z in the middle.

Choose the following labeling for the basis

$$|n_1, n_2, n_3 \ldots \rangle \simeq | \uparrow, \downarrow^{\otimes n_1}, \uparrow, \downarrow^{\otimes n_2}, \uparrow, \downarrow^{\otimes n_3}, \ldots \rangle$$

Can do same for closed strings

After some work we can show that the 1-loop anomalous dimension (spin 1/2 chain) for bulk is given by a nearest neighbor interaction

$$H_{eff} = g_{YM}^2 N \sum_i (a_{i+1}^\dagger - a_i^\dagger)(a_{i+1} - a_i)$$

Cuntz oscillator at each site

In a bosonic basis.

Which clearly shows it is a sum of squares.

Ground states?

Cuntz oscillators

$$aa^\dagger = 1$$

After some work ... boundary terms can be computed

Still a sum of squares

$$H_{eff} \simeq g_{YM}^2 N \left[\left(\frac{\lambda}{\sqrt{N}} - a_1^\dagger \right) \left(\frac{\lambda^*}{\sqrt{N}} - a_1 \right) + (a_1^\dagger - a_2^\dagger)(a_1 - a_2) + \dots \right]$$

D.B., E. Dzienkowski arXiv:1305.2394

To find ground state, coherent state ansatz

$$\langle z_1, \dots z_k | H_{\text{spin chain}} | z_1, \dots z_k \rangle = g_{YM}^2 N \left[\left| \frac{\lambda^*}{\sqrt{N}} - z_1 \right|^2 + \sum |z_i - z_{i+1}|^2 + \left| \frac{\tilde{\lambda}^*}{\sqrt{N}} - z_k \right|^2 \right]$$

and minimize

$$\frac{\lambda^*}{\sqrt{N}} - z_1 = z_1 - z_2 = \cdots = z_i - z_{i+1} = \cdots = z_k - \frac{\tilde{\lambda}^*}{\sqrt{N}}$$

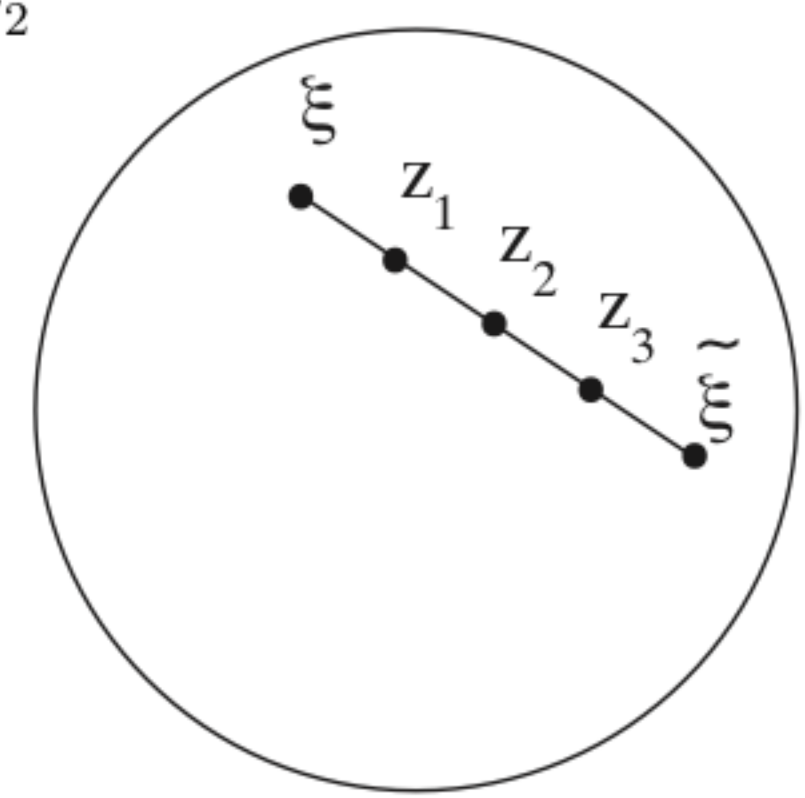
We can add these to solve the linear equations

$$\frac{\lambda^*}{\sqrt{N}} - \frac{\tilde{\lambda}^*}{\sqrt{N}} = (k+1)(z_i - z_{i+1})$$

$$E_0 = \frac{g_{YM}^2 N}{k+1} \left| \frac{\lambda}{\sqrt{N}} - \frac{\tilde{\lambda}}{\sqrt{N}} \right|^2$$

$\xi = \lambda^* N^{-1/2}$

$\tilde{\xi} = \tilde{\lambda}^* N^{-1/2}$



These can be pictured on the “free fermion disk”

The z coordinates also have a geometric interpretation!

$$E(z_0, \ldots, z_{k+1}) \simeq g_{YM}^2 N \sum |z_{i+1} - z_i|^2$$

Full two loop result

$$= \sum_{l=1}^{L-1}(a_{l+1}^\dagger - a_l^\dagger)^2(a_{l+1}-a_l)^2 + \left(a_1^\dagger - \frac{\lambda}{\sqrt{N}}\right)^2\left(a_1 - \frac{\bar\lambda}{\sqrt{N}}\right)^2 + \left(a_L^\dagger - \frac{\tilde\lambda}{\sqrt{N}}\right)^2\left(a_L - \frac{\bar{\tilde\lambda}}{\sqrt{N}}\right)^2$$

$$+ \sum_{l=2}^{L-1}(a_{l+1}^\dagger - 2a_l^\dagger + a_{l-1}^\dagger)[a_l, a_l^\dagger](a_{l+1} - 2a_l + a_{l-1})$$

$$+ \left(a_2^\dagger - 2a_1^\dagger + \frac{\lambda}{\sqrt{N}}\right)[a_1, a_1^\dagger]\left(a_2 - 2a_1 + \frac{\bar\lambda}{\sqrt{N}}\right)$$

$$+ \left(\frac{\tilde\lambda}{\sqrt{N}} - 2a_L^\dagger + a_{L-1}^\dagger\right)[a_L, a_L^\dagger]\left(\frac{\bar{\tilde\lambda}}{\sqrt{N}} - 2a_L + a_{L-1}\right)$$

=0 in ground state

From the gravity side

Need to modify a calculation in sigma model on a three sphere times time.

H. -Y. Chen, N. Dorey and K. Okamura, "Dyonic giant magnons," JHEP 0609, 024 (2006) [hep-th/0605155]

Chrysostomos Kalousios, Marcus Spradlin, and Anastasia Volovich,JHEP, 0703:020, 2007

Final answer is

$$\Delta - J = \sqrt{J_2^2 + \frac{\lambda}{4\pi^2}|\xi - \tilde{\xi}|^2}$$

D.B., E. Dziekowski arXiv:1408.3620

Why? Central charge extension

Acting on a Y

$$Y \to [Z, Y]$$

Beisert hep-th/0511082

in Cuntz basis

$$\sqrt{N}(a_i^\dagger - a_{i+1}^\dagger)$$

OR

$$Y \to [Y, \partial_Z]$$

$$(a_i - a_{i+1})/\sqrt{N}$$

And remember that our ground states are eigenstates of these lowering operators. It gives

$$z_i - z_{i+1}$$

Total central charge

$$\mathfrak{C} = \sum (z_i - z_{i+1}) = z_0 - z_n = \xi - \tilde{\xi}$$

independent of the state, but sourced by D-branes

Small representation of centrally extended PSU(2|2)

$$E = \sqrt{n^2 + g^2 N |\xi - \tilde{\xi}|^2}$$

Exact result to all orders

We had 1 and 2-loop verified

Now deform N=4 SYM

$$W \simeq \mathrm{Tr}(XYZ - qXZY)$$

Leigh-Strassler

Special case

$$qq^* = 1$$

Preserves integrability

$$q = \exp(2i\beta)$$

$$H_{1-loop} = \sum (a_i^\dagger - q^* q_{i+1}^\dagger)(a_i - q a_{i+1})$$

The q can be removed by twisting (D.B + Cherkis, hep-th/0405215)

This effectively changes

$$\tilde{\xi} \to \tilde{\xi} q^n$$

$$E = \sqrt{n^2 + g^2 N |q^{-n/2}\xi - q^{n/2}\tilde{\xi}|^2}$$

Dispersion relation, which is relativistic + something that looks like a lattice dispersion relation.

Geometric limits: “lots of operators with small anomalous dimensions”

You have a lot of supergravity and field theory modes on branes that do not become stringy, rather, effective field theory on a SUGRA background.

Simplest one

$$q^k = 1 \quad + \quad g^2 N \to \infty$$

$$+ \quad g^2 N |\xi - \tilde{\xi}|^2 \quad \text{fixed or scaled}$$

Only n=km survives at low energies

This indicates a theory on giants of the form

$$S^3/\mathbb{Z}_k$$

We can now consider also “images”

$$\tilde{\xi} = \xi q^s$$

We recover light modes when

$$n = -s \mod k$$

Indicates a relative Wilson line on the quotient sphere.

Another limit, small beta

$$E \simeq \sqrt{n^2 + g^2 N|\xi - \tilde{\xi} - \xi i\beta n + \tilde{\xi}(i\beta)n + \ldots|^2}$$

Now take

$$\xi = \tilde{\xi}$$

$$E \simeq \sqrt{n^2 + g^2 N|\xi|^2 \beta^2 n^2}$$

Is of order n if

$$g^2 N \beta^2 \simeq 1$$

Interpretation

$$E \simeq An$$

Think about this as the spectrum of a relativistic particle on a circle

$$A \simeq \frac{1}{R(\xi)} = \sqrt{1 + |\xi|^2 g_{YM}^2 N \beta^2}$$

We start seeing cycles getting squashed

Another limit, small beta

$$E \simeq \sqrt{n^2 + g^2 N|\xi - \tilde{\xi} - \xi i\beta n + \tilde{\xi}(i\beta)n + \ldots|^2}$$

Now take

$$\xi = \tilde{\xi}\exp(-2i\theta)$$

$$E \simeq \sqrt{n^2 + g^2 N|\xi|^2\beta^2(n+\theta/\beta)^2}$$

When we complete the square, we get a "position dependent Wilson line"

This has to be interpreted as the

$$H_{\mu\nu\rho}$$

Field strength in gravity.

Dual giants

$$H_{eff} \simeq g_{YM}^2 N \left[\left(\frac{\lambda}{\sqrt{N}} - a_1^\dagger \right) \left(\frac{\lambda^*}{\sqrt{N}} - a_1 \right) + (a_1^\dagger - a_2^\dagger)(a_1 - a_2) + \dots \right]$$

Same Hamiltonian! now the collective coordinates end up outside the unit disk.

DB: arXiv:1411.5921

Generically BPS ground state in SU(2) sector does not exist, except for 0 sites (1 Y).

The z coherent state coordinates end up outside the unit disk: they don't belong to the Hilbert space of the chain (non-normalizable states).

One can cross walls of marginal stability.

Can take limits

Large collective coordinates for D-branes land us on Coulomb branch. If we scale effective masses to be very large, radius of 3-sphere becomes irrelevant.

We also need to decouple strings.

$$1 << g_{YM} \sqrt{N} |\mathcal{Z}_1 - \mathcal{Z}_2| << \ell_s^{-1} \cosh \varrho = (g_{YM}^2 N)^{1/4} |\mathcal{Z}|$$

In the limit

Beisert's central charge evaluates to the Coulomb branch central charge of N=4 SYM.

Beisert's central charge contains the other one as a limit.
This probably explains the non-renormalization of h rigorous

It also relates the two shortening conditions of BPS states.

Conclusion

- Ground state of open strings between (dual) giant gravitons is soluble (2-loop and sigma model)
- The giant gravitons source Beiserts central charge and energy of state is controlled by SUSY.
- Can do beta-deformations (can see details of geometry)

- Beisert's central charge contains the central charge of the Coulomb branch of N=4 SYM.

- Can play similar games with N=2 SUSY (no integrability, but central charge available — Gadde,Pomoni,Rastelli)